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Final Report



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12/12/2021

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Dear Dr. Romanyk,

Koopa Auto Group (KAG) is pleased to present the Final Report displaying the detail considered in the analysis of the proposed transmission. KAG has dedicated the past four months to the design and analysis of a transmission, developing a comprehensive solution to the problem presented by MecE 360 Inc.

In this report, KAG outlines the design methodology used to select the final design and provides the detailed analysis for the shaft, gears, and bearings selected. This analysis was presented during the oral presentation given but will be elaborated upon in the analysis section of this report. KAG produced a solid model of the total transmission, providing a final drawing package for proper dimensioning of the design.

KAG would like to thank MecE 360 Inc. for the opportunity to collaborate on such a project. This project has given our team members a great deal of insight when it comes to the nature of transmissions and strengthened our abilities as future engineers, with all of our employees listed below.

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All Intellectual Property (IP) will be owned by MecE 360 Inc.

Sincerely,



CORREIA, Nathan PARISH, Arrey LAM-TRAN, Katherine MANZ, Joel NGUY, Quinton ZHI, Xin Wei

ABSTRACT

The following report describes Koopa Auto Group's (KAG) current design of the 4-speed transmission, as contracted by MecE 360 Inc. that would be sufficient for Dry Bones to race with. The transmission includes an input and output shaft, with each having four gears that were selected using calculated gear ratios held by bearings on the ends of each shaft. The lifetime of the transmission was assumed to be 4 races or about 10 hours. This report outlines the design of the shafts, gears, and bearings to fall within evaluation criteria and the design process.

The input and output shaft will be made from AISI 1080 Steel. The minimum diameter was calculated using ASME DE Elliptic criterion, and it was found that the bore diameter of 42 mm for the input shaft and 43 mm for the output shaft was sufficient by being larger than the ASME DE Elliptic diameter found. Angular deflection at the gear and bearings, total deflection at the gears, and the angle of twist all fall within the specified evaluation design criteria. Through this analysis, the diameter at the bearings will be 35 mm for the input shaft, and 40 mm for the output. The keys used for mounting the gears onto the shaft were also analyzed for failure and were found to be within acceptable values for the material chosen.

A full gear analysis has been completed with the gear ratios between the first and fourth gears being 0.52, 0.80, 1.25, and 1.94, respectively. These ratios were found after the final drive ratio was calculated to be 0.194, allowing for KAG to reduce the ratios that were initially found. The gears were selected from RushGears based on simplified ratios of 0.5, 0.8, 1.25, and 2.

Due to the high speeds involved, angular contact ball bearings were the chosen type due to their high limiting speeds. Fatigue life and critical force is 7.8 million cycles and 1279.48N as well as 3.6 million cycles and 1612.04N were obtained for the output and input shaft, respectively. Bearing number 7908 was chosen for the output and bearing 7907 number for input shaft and were selected from the NSK bearing catalogue.

Utilizing AGMA standards, assumptions, and equations, KAG was able to compute the bending and contact stresses for each individual gear. The selected gears for KAG's transmission system have been calculated to successfully transfer engine torque to the driveshaft without failing as all calculated bending and contact stresses are less than the calculated bending and surface strength.

Table of Contents

Abstract	2
List of Figures	4
List of Tables	4
Introduction.....	5
Design Methodology And Project Planning	6
Concept Selection	6
Analysis.....	8
Shaft Analysis	8
Keyway Analysis	10
Gear Analysis.....	11
Gear Ratio Analysis	11
Gear Force Analysis.....	12
Shifting Mechanism	14
Summary of Final Design	15
Conclusion	15
References.....	16
Appendix A: Letter of Intent.....	17
Appendix B: Gantt Chart	18
Appendix C: Mind Map	19
Appendix D: Decision Matrices.....	20
Appendix E: Shaft Analysis.....	21
Appendix F: Gear Analysis.....	48
Appendix G: Bearing Analysis	57
Appendix H: Drawing Package	58
Appendix I: Meeting Minutes.....	66

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LIST OF FIGURES

Figure 1: Finalized shaft design, created in SolidWorks (see Appendix H).....	7
Figure 2: Free Body Diagram (FBD) of Input Shaft.....	8
Figure 3: Course #2 with wide turning radii and long straits	11
Figure 4: Assembly View of shifting mechanism on the input shaft, created in SolidWorks (see Appendix H).....	15
Figure 5: Gantt Chart used for project deadlines	18
Figure 6: KAG Go Kart Transmission Mind Map.....	19
Figure 7: KAG Drawing Tree	58

LIST OF TABLES

Table 1: Summary of values found using SMath for the input and output shaft	10
Table 2: Summary of Fatigue and Yield Safety Factors based on the gear selected	10
Table 3: Gear Force Analysis results	13
Table 4: Bending and Contact Safety Factors calculated for each gear.....	13
Table 5: Decision Matrix to select 4 or 5 Speed Transmission	20
Table 6: Decision Matrix to select Dog Clutch or Synchronizers	20

INTRODUCTION

Koopa Auto Group (KAG) has been contracted by MecE 360 Inc. to design a transmission system for a Mario Kart Go-Kart. Specifically, a 4 or 5 speed manual transmission that is to be powered by a Kawasaki KLR650 CAMO engine with a maximum engine output value of 24.4 kilowatts at 6500 rpm. It has been specified that the primary reduction will be 2.272. MecE 360 Inc. has specified the use of stock gears and bearings from manufacturers only. Finally, the gearbox must take the output torque, speed, and weight of the go-kart along with the chosen rider. Koopa Auto Group has designed a 4-speed transmission with a dog clutch shifting mechanism without reverse gearing. KAG chose Dry Bones as the driver to compete on Course #2, Luigi Circuit whose dimensions will be based on the Monza circuit by Formula 1. The calculated combined mass of Dry Bones and the go-kart is 218 kg. KAG has done most calculations under the condition that the primary reduction is no longer used, making the transmission more robust and versatile. KAG has utilized all design standards in major calculations such as ISO and AGMA standards for spur gears, and ASME for strength criteria. The group has taken all pre-set project specifications into consideration throughout all aspects of the project lifecycle.

DESIGN METHODOLOGY AND PROJECT PLANNING

In the early phases of the project, emphasis was placed on organization to allow for a more stream-lined work process. The most prominent example is the Gantt chart (Appendix B), which laid out a long-term plan for design methodology and workflow by specifying dates and times to work on each portion of the project. Regular weekly meetings were also set up and meeting minutes were taken regularly to ensure consistency between schedules and assigned tasks (Appendix I).

KAG also made use of different methods for stream-lining concept creation and selection. Initially a mind map was used for organization of initial ideas (Appendix C). These initial ideas were then brainstormed, and a few potential candidates were chosen for the final design. Namely, a four speed with dog clutch shifting mechanisms, five speed with dog clutch shifting mechanisms and reverse as well as a four speed with synchronizer shifting mechanisms. The ideas were then evaluated using weighted criteria in design matrices where KAG determined the optimal design to be a four-speed with dog-clutch shifting mechanisms.

CONCEPT SELECTION

During the concept selection phase, a mind map was created (Appendix C) to decide between initial designs. From this, three initial concepts were considered: a 4-speed speed transmission with synchronizers, a 4-speed transmission using a dog clutch, or a 5-speed transmission using a dog clutch. These concepts all involved spur gears due to their greater availability.

Based on the initial concepts, decision matrices (Appendix D) were created to compare a 4-speed or 5-speed transmission, as well as a synchronizer and dog clutch. The matrices examined the strengths and weaknesses of the designs, using the factors shown below:

For the transmission:

Compactness (10): The size of the transmission with gears and components. Space within a go-kart is limited and creating a compact transmission will reduce the overall mass of the vehicle.

Efficiency (10): How fast the kart will accelerate and its shift recovery time. This was given a weight of only ten due to the track selected which consists of mostly long straits, as the kart is expected to reach top speed and maintain that speed for extended periods of time without many gear shifts.

For the clutch:

Quickness (50): The time required to shift between gears in sequential order. It is vital that Dry Bones can quickly shift between gears when turning corners and entering new straits, while remaining as close to the maximum power output as possible.

For both:

Reliability (15): Ability of the transmission to function under specified conditions for a specified period without failure. This is to identify the chance of failure during a race, while cost is not an issue it is important that the driver remain safe.

Weight (20): The mass of the transmission or shifting mechanism. Less energy will be required to accelerate a lighter go-kart than a heavier one, meaning that the mass should be optimized. A lower weight involved will aid in creating a more efficient transmission.

Durability (5): Durability has been defined as the transmission's predictable lifespan. The kinds of wear and tear those parts may experience during the race, and the lifetime of the parts. Durability was weighted relatively low as KAG plans on recreating the transmission after a certain number of races to maximize the quality of the transmission.

Cost (5): The total cost of the object. MecE 360 Inc. stated that cost was not a large factor, but KAG decided to include it at a lower weighting as there would be cost associated with overhauling the transmission after a determined number of races.

The completed decision matrices closely selected the four-speed transmission with a dog clutch, mostly due to the transmission's reliability and compactness, as well as the dog clutch's quickness. This design is what KAG ultimately decided upon using.

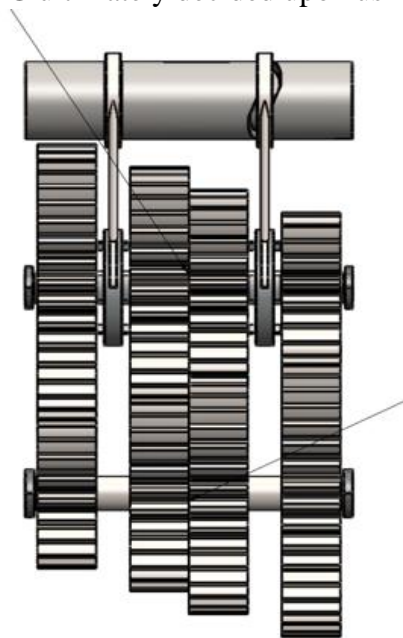


Figure 1: Finalized shaft design, created in SolidWorks (see Appendix H)

ANALYSIS

Shaft Analysis

These shafts were designed to tolerate the forces that result from the bearings and gears and will be able to transmit torque from the engine. To begin this process, KAG made assumptions that would make the initial steps easier to compute.

1. Gears used are spur gears
2. Shafts are designed with 90% reliability and infinite life, with no miscellaneous factors
3. Shafts are machined
4. Bending is dominant on both shafts
5. Torque application is constant
6. The shaft is supported by two bearings at the ends
7. The bearing and gear forces act as point loads
8. All dimensions and sizes fit within the normal range of values
9. There is no undercutting or interference of the gears
10. There is a safety factor of 1

It is important to note that the only forces that will be acting upon the shaft at any given moment will depend on the gear selected. As seen in the Free Body Diagram (FBD) below, the tangential and radial forces of only one gear will act upon the shaft. These values are determined using the pitch diameter of each gear, and the distance from the left bearing will be important to determine the reaction forces. To make calculations easier, KAG created an SMath program that has the user select a specific gear (Gear 1, 2, 3, or 4) which assigns the distance from the left bearing, the pitch diameter, and number of teeth for the input shaft (Appendix E). For the output shaft, using the same system allows the user to select the same criteria listed previously and includes the gear ratio to alter the torque experienced by the shaft (Appendix E).

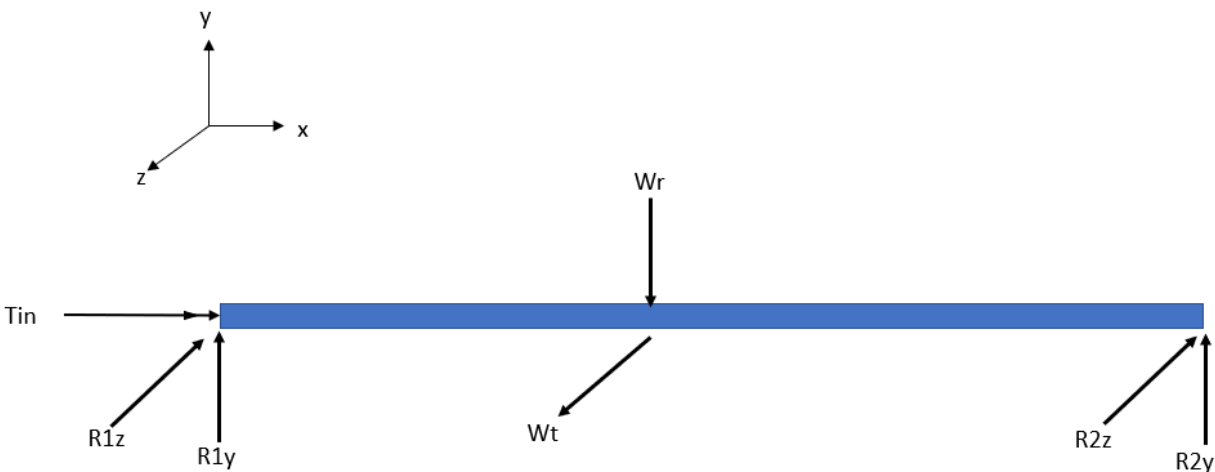


Figure 2: Free Body Diagram (FBD) of Input Shaft

To space the gears apart, an initial shaft length was assumed to be 425 mm. After deciding that the shaft ends would be at the center of the bearings, KAG initially spaced the gears evenly apart. After the Analysis Report was submitted, it was decided to shift gears 2 and 3 so that they would remain directly next to each other to reduce the time it would take to shift between the two gears. When creating the solid model, it was found that there would not be much space between gears 1 and 2, and gears 3 and 4 which would be needed to allow a dog clutch ample space to move. To accommodate this, gears 1 and 4 were shifted closer roughly 20 mm to the left and right bearings, respectively.

The gear ratios used in calculation were taken as 2, 1.25, 0.8, and 0.5 for gears 1 through 4, respectively. The process for calculating the gear ratios can be seen later in the document under ‘Gear Ratio Analysis’.

Initially, AISI 1020 Steel was used, but this caused too much deflection at the gears, so AISI 1080 Steel was selected because of its higher modulus of elasticity [1]. As well, temperature needed to be considered and was taken as 195 F [2]. Using AISI 1080 Steel, KAG used singularity functions to produce moment diagrams, eventually leading to the calculation of stress concentration factors (SCFs), as well as endurance strength of the shaft [3] [4].

While there are steps in the shafts with a step radius of 3 mm, keyways proved to be important for the SCFs which would later be used to calculate the minimum diameter using the ASME DE Elliptic equation. It was assumed that the keyways would contribute to the stress concentration factors the most, so their respective K_t and K_{ts} values were used to find K_{fm} and K_{fms} .

Using the ASME DE Elliptic equation, a minimum diameter was found for the shaft at each gear shift. This diameter was not taken directly as the diameter of the bore shaft. While it passed using the assumption of a minimum safety factor of 1, these values were found to cause unacceptable deflection and twist when compared to the evaluation criteria previously mentioned. Thus, a bore diameter of 42 mm for the input shaft was taken to allow for a more space for a keyway while a bore diameter of 43 mm was taken for the output shaft. It was initially found that a bore diameter of 38-39 mm would suffice, but it was difficult to find gears with that specific diameter, so the bore diameter was increased to match the bore diameter of the gears selected. The diameter at the bearings will be 35 mm for the input shaft and 40 mm for the output shaft.

To ensure the safety of the shaft, KAG followed ASME DE elliptic strength criteria which limit the slope, deflection, and twist of the shaft.

- Deflection at gears < 0.127 mm
- Angular deflection at gears < 0.0005 rad
- Angular deflection at bearings < 0.004 rad
- Angle of twist < 3 deg/m

The results of KAGs analysis can be found in Table 1.

Table 1: Summary of values found using SMath for the input and output shaft

Shaft	Gear Selected	Minimum Diameter (mm)	Maximum Deflection at the Gears (mm)	Maximum Angular Deflection at the Gears (rad)	Maximum Angular Deflection at the Bearings (rad)	Maximum Angle of Twist (deg/m)
Input	1	15.63	0.028	0.0005	0.0009	0.191
	2	17.54	0.080	0.0002	0.0009	0.191
	3	18.40	0.087	$7.43 \cdot 10^{-5}$	0.0009	0.191
	4	16.74	0.021	0.0003	0.0005	0.191
Output	1	16.74	0.022	0.0004	0.0006	0.174
	2	18.40	0.050	0.0001	0.0005	0.174
	3	17.54	0.037	$4.74 \cdot 10^{-5}$	0.0003	0.174
	4	15.63	0.0061	$9.40 \cdot 10^{-5}$	0.0001	0.174

While it was assumed that the minimum safety factor was 1, KAG moved forward with calculating the fatigue safety factor and yield safety factor using the bore diameter of the shaft. The findings have been summarized in the table below.

Table 2: Summary of Fatigue and Yield Safety Factors based on the gear selected

Shaft	Gear Selected	Fatigue Safety Factor	Yield Safety Factor
Input	1	19.39	22.28
	2	13.73	17.91
	3	11.89	16.27
	4	15.13	19.07
Output	1	16.94	21.03
	2	12.76	17.46
	3	14.74	19.22
	4	20.80	23.91

Keyway Analysis

It was decided that keyways would be used to mount the gears onto the shaft, meaning that the keyways needed to be analyzed for potential failure as they would be SCFs on the shaft. KAG will be using stainless steel for the keys and using the maximum torque on both the input and output shaft for analysis. The keys on the input shaft would be experiencing 81.44 Nm of torque with this translating to be 2.12 MPa of stress. The key experiencing the most torque on the output shaft would be the one at the first gear (162.89 Nm) and would experience a stress of 4.24 MPa. These keys are well under the maximum stress that stainless steel can experience, which is 74.5 MPa (see Appendix E).

Gear Analysis

Gear analysis was divided into two major sections, gear ratio analysis and gear force analysis. The gears utilized in the transmission have been optimized for maximum possible speed around Course #2, and they have been designed to withstand the necessary forces to transmit torque from the engine to the wheels.

Gear Ratio Analysis

Given a primary reduction of 2.272, the optimal gear ratios were determined by KAG through analysis of the track, and its features such as straightaways, high radius turns, and low radius turns. To determine the features of course #2, estimates were used based off the course's real-life inspiration, Monza circuit. Through research it was found that a typical go-kart-track is approximately 1/6 the length of a racing track [5] [6]. Using this factor and dimension for Monza circuit, the radii of turns as well as the length of straightaways were estimated [7].

The most important values found were the maximum and minimum speeds achieved on track as these values allowed for the initial and final gear ratios to be determined. The maximum speed on track was found at the end of the longest straightaway and the minimum speed was found at the smallest radius corner and were found to be approximately 130km/h and 50km/h respectively using a coefficient of drag of 0.5 [8] (see Appendix F).

With 1st and 4th gear determined, 2nd and 3rd gear followed. They were calculated by allowing a common multiple to exist between each gear ratio (see Appendix F, final_gear_calcs.sm). This causes a common loss of RPMs when shifting up for all gears which is ideal for staying as close to the peak horsepower of the engine as possible. The calculated gears were found to be 2.64, 4.10, 6.37, and 9.89, and the common ratio between them was 1.55. This means that each shift causes a 1/1.55 or 34% loss in RPMs. From peak RPM this leaves us at 4900 RPM corresponding to about 31hp [9].

With each gear ratio determined KAG found that the most extreme gear ratio of 9.89 was difficult to work around. This was because higher gear ratios are generally less efficient and most importantly it created a large distance between the shafts which increased the magnitude of the forces that the gears placed on the shafts. As such, a final gear ratio was placed between the wheels and the transmission system. This allowed extreme gear ratios such as the 4th gear to be minimized. The geometric mean of the 1st and 4th gear was found to be 5.1, so was used as the final gear ratio. This is because dividing each of the transmission gear ratios by 5.1 led to 1st and 4th gear being reciprocals of each other, which minimizes gear ratio extremity. Any deviation from this final gear ratio would either cause 1st gear to become more extreme or 4th gear to become more extreme.



Figure 3: Course #2 with wide turning radii and long straits

The ratios then became 0.52, 0.80, 1.25, and 1.94. This design of final drive ratio also had the benefit of 1st and 4th as well as 2nd and 3rd gears being the same gear pairs. This allowed KAG to have a more streamlined gear selection process and allowed for more variation in potential gear candidates.

Finally, the values were rounded to simple ratios to allow for gear selection to occur. The transmission ratios were approximated as 0.5, 0.8, 1.25, and 2, while the final drive ratio was approximated as 5. The error in top speed from this approximation was calculated and found to be 5.6% error, which was within KAG's margin of acceptance (Appendix F).

Gear Force Analysis

Stresses on the gears are induced by the torque generated from the Kawasaki KLR 650 CAMO engine used in the go-kart. Tangential forces are a result of this and are imposed onto the gears; bending and contact stresses arise. AGMA equations and assumptions for spur gears were used in the approximation of these stresses along with the following KAG assumptions:

1. Gear quality index is a 9
2. Gears undergo moderate shock from a 4 stroke, single cylinder engine
3. Gears are a solid disk with no rims
4. Operating temperature is 195 degrees Fahrenheit
5. Gears are designed with 90% reliability
6. Transmission is designed for 107 cycles

All gears models in addition to crucial gear information such as gear characteristic values (face width, pitch diameter etc.) were found on the manufacturer's website [10]. To ensure the chosen gears can withstand the forces that will be placed upon them, KAG calculated the bending and contact stresses of each individual gear, along with their corresponding strengths. An SMath code was developed to calculate these forces, stresses, and safety factors (see Appendix F, MECE360_Gear_Analysis_UPDATED.sm).

All factors obtained and utilized in calculations (geometry, application, load distribution etc.) were obtained using AGMA tables, charts and values found on the official AGMA standards manual [11][12].

The results of KAG's gear force analysis can be found in Table 3.

Table 3: Gear Force Analysis results

	Gear Selected	Bending Stress (MPa)	Contact Stress (MPa)	Bending Strength (MPa)	Surface Strength (MPa)	Gear Model
Input	1	23.4	314.3	1045.8	1908.5	F342
	2	26.5	316.9	1048.8	1908.5	F335
	3	32.0	341.9	1080.9	1932.5	F328
	4	40.8	398.9	1158.8	1984.0	F321
Output	1	25.4	444.5	1045.8	1908.5	F321
	2	27.5	354.3	1045.8	1908.5	F328
	3	30.8	305.8	1080.9	1932.5	F335
	4	37.6	281.6	1158.8	1984.0	F342

Although assumed to be 1.0, KAG calculated the bending and wear safety factors. The values can be found below.

Table 4: Bending and Contact Safety Factors calculated for each gear

	Gear Selected	Bending Safety Factor	Contact Safety Factor
Input	1	44.6	36.9
	2	39.5	36.3
	3	33.7	31.9
	4	28.4	24.8
Output	1	41.2	18.4
	2	38.0	29.0
	3	35.1	39.9
	4	30.8	49.6

It should be noted that the critical gear in this transmission system is output gear 1 as it has the lowest safety factor at 18.4. The lowest safety factor falls under contact safety factors, and thus input gear 4 would be expected to fail via contact fatigue before any other gear.

Bearing Analysis

Bearings were placed on either end of both shafts, to hold the shaft in place and minimize friction during shaft rotation. Since spur gears are used in the transmission, the force on the shaft and therefore the reaction forces on the bearings were assumed to be purely radial. These forces were taken as the maximum reaction forces resulting from shaft analysis. Due to KAG's plan to replace the transmission every four races, a lifetime of 10 hours was assumed. For the calculation of fatigue life, the speed for the input shaft was assumed to be equal to the maximum speed of the Kawasaki KLR650 engine, which is 6,500rpm. The speed of the output shaft was then assumed to be the resulting speed when operating at the highest gear ratio, which resulted in a speed of 13,000rpm. Due to these high speeds, angular contact ball bearings were the chosen type due to their high limiting speeds. The bearings will be lubricated with oil, allowing for higher limiting speed compared to grease.

With the values for speed and lifetime, fatigue life of 7.8 million cycles and 3.6 million cycles were obtained for the output and input shaft, respectively. Using the calculated fatigue life, reaction forces, 10% survival probability (reliability factor of 1), and assumption of ball bearings, the critical force for input bearings was calculated to be 1612.04N, and the critical force for the output bearings was calculated to be 1279.48N (see Appendix F).

Using the calculated critical forces for the input and output shafts and the assumed operating speeds, bearing number 7908 and 7907 were chosen for the output and input shafts, respectively. The basic dynamic load ratings for 7908 and 7907 is 14,300N and 11,400N respectively. Since the ratio of the expected radial load to dynamic load rating is approximately 8 for both gears, the uncorrected limiting speed for the bearings was multiplied by a correction factor of 0.9, in accordance with NSK's rolling bearing correction of limiting speed [13]. After accounting for correction due to loading, the limiting speed of bearing 7908 is 15,300 rpm, and the limiting speed of bearing 7907 is 11,400rpm [13].

Shifting Mechanism

A sequential shifting mechanism was chosen by KAG over a traditional H-pattern gear-box due to its potential for faster shifting in race applications. This involved the rotation of a 'shifting shaft' to move the dog clutches. Shifting links were used to connect the shifting shaft and the dog clutches and the movement of the dog clutches was achieved using grooves on the shaft and inserts on the shifting links. To create the 4-speed transmission, two dog clutches were needed, two shifting links and two grooves on the shifting shaft were also necessary. The grooves on the shifting shaft were designed to ensure that the transmission could only be in a single gear at a time and allowed for a neutral position as well. The grooves were also designed in such a way that 1/5 of a rotation would cause a change by one gear.

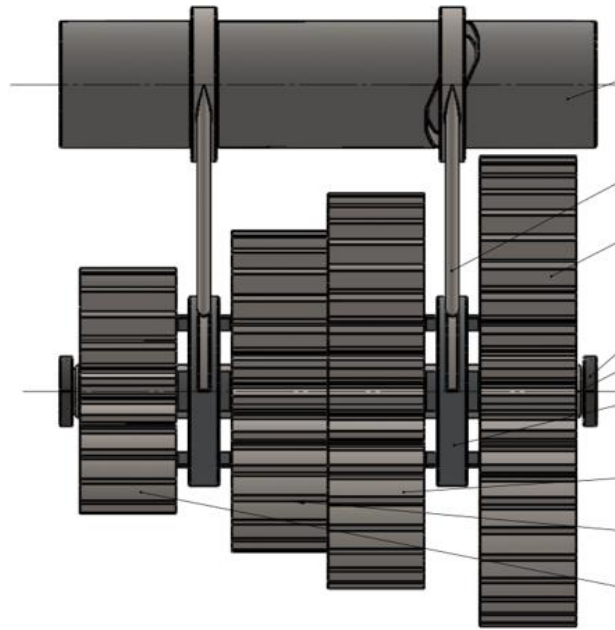


Figure 4: Assembly View of shifting mechanism on the input shaft, created in SolidWorks (see Appendix H)

SUMMARY OF FINAL DESIGN

KAG was able to produce 4-speed transmission using a dog clutch with gear ratios of 0.5, 0.8, 1.25, and 2. The input shaft will have a bore diameter of 42 mm, a bearing diameter of 35 mm with a step radius of 3 mm. For the output shaft, there will be a bore diameter of 43 mm and a bearing diameter of 40 mm with the same step radius as the input shaft. Gears were selected from RushGears and the bearings were taken from the NSK bearing catalogue. KAG created detailed drawings to outline key measurements required when manufacturing and machining the various parts included (see Appendix H).

CONCLUSION

In this final report, as contracted by MecE 360 Inc., KAG was able to design a transmission for the go-kart as driven by Dry Bones along Course #2 which has been modelled after the Monza Circuit from Formula 1. To begin analysis, the gear ratios were calculated based on top speed calculations, and an in-depth force analysis was completed for each gear. Once the gear ratios were calculated, the shaft analysis was done, calculating the ASME DE elliptic minimum diameter and calculating to see if there would be too much deflection or twist along the shaft regarding the ASME evaluation criteria. The ASME DE Elliptic diameter aided in the selection of gears that would then need to be analyzed. Bearing calculations were done to select bearings that would be able to withstand the torque on the shaft as a final step. KAG was also able to design a dog clutch for the transmission as the shifting mechanism. KAG is confident that Dry Bones will be able to successfully navigate Course #2 with the transmission designed and will come first in the race.

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APPENDIX A: LETTER OF INTENT

Koopa Auto Group
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September 10, 2021

Prof. Dan Romanyk
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Dear Dr. Romanyk,

Koopa Auto Group (KAG), formerly Group #12, is excited to present this Letter of Intent regarding the Mario Kart Transmission Project. KAG will be designing the transmission system to be optimized for the character Dry Bones, and course #2.

KAG understands the scope of work that this project entails and agrees to all conditions specified by MecE360 Inc., as described in the file "MecE360 Project Syllabus" (Accessed: 09/08/2021).

The seven currently contracted employees of KAG and their contact information are listed in the table below:

Name (Last, First)	Student ID Number	Email
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LAM-TRAN, Katherine	1615374	lamtran@ualberta.ca
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ZHI, Xin Wei	1616941	xinwei@ualberta.ca

Moreover, some important dates associated with the availability of project information to the client are:

Project Information	Date
Letter of Intent	09/12/21
Concept Report	10/10/21
Analysis report	11/07/21
Final Report	12/08/21

The above information will be sent to the client electronically via the online program, eClass.

Our team leader and company contact will be Nathan Correia (ncorreia@ualberta.ca), who will speak on behalf of the group regarding the project's progress at any given time.

Furthermore, if the bid is accepted, all design intellectual property (IP) will remain that of the University of Alberta's MecE360 Inc. Thank you for considering Koopa Auto Group, we look forward to working together.

Sincerely,

CORREIA, Nathan BASHIR, Clara PARISH, Arrey LAM-TRAN, Katherine MANZ, Joel NGUY, Quinton ZHI, Xin Wei

APPENDIX C: MIND MAP

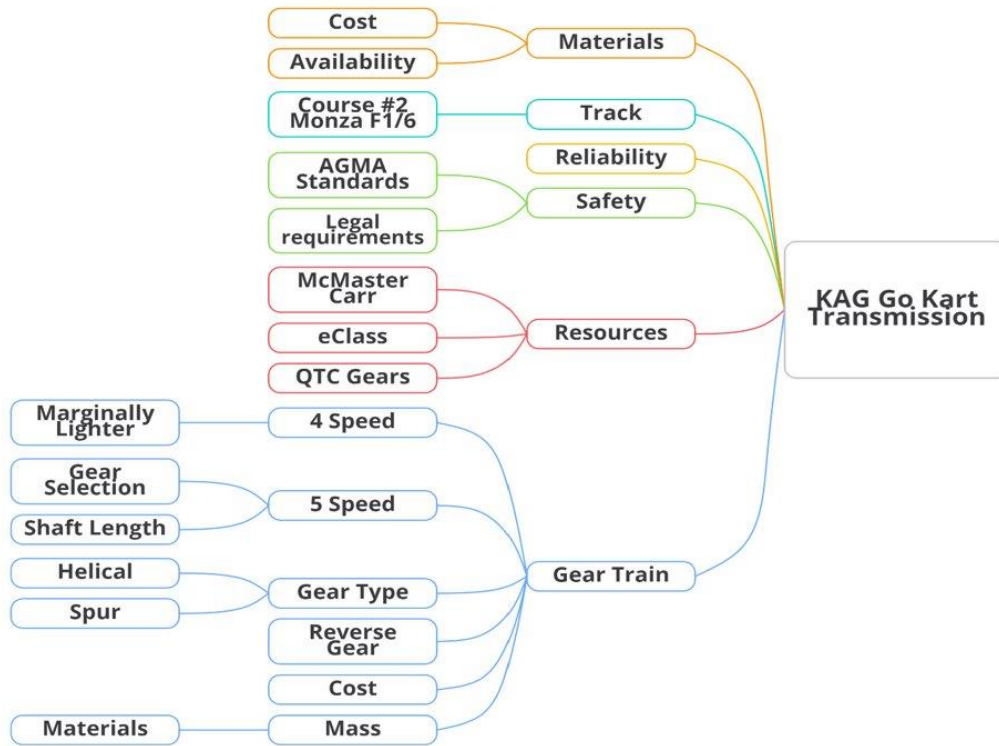


Figure 66: KAG Go Kart Transmission Mind Map

APPENDIX D: DECISION MATRICES

Table 5: Decision Matrix to select 4 or 5 Speed Transmission

4 or 5 Speed							
Criteria							
	Reliability	Compactness	Efficiency	Weight	Durability	Cost	
MAX VALUE	10	10	50	20	5	5	Total
4 Speed	8	9	40	18	5	5	85
5 Speed	5	7	45	18	4	4	83

Table 6: Decision Matrix to select Dog Clutch or Synchronizers

Shifting Mechanism							
Criteria							
	Reliability	Quickness	Weight	Durability	Cost		
MAX VALUE	15	50	25	5	5		
Dog Clutch	8	50	20	4	5	87	
Synchronizers	12	30	20	5	4	71	

APPENDIX E: SHAFT ANALYSIS

KAG Input Shaft Analysis

KAG Input Shaft Analysis

☐ Assumptions, Evaluation Criteria, Torque Calculations

Assumptions:

1. Gears used are spur gears
2. Shafts are designed with 90% reliability and infinite life, with no miscellaneous factors
3. Shafts are machined
4. Bending is dominant on both shafts
5. Torque application is constant
6. The shaft is supported by two bearings at the ends
7. The bearing and gear forces act as point loads
8. All dimensions and sizes fit within the normal range of values
9. There is no undercutting or interference of the gears
10. There is a safety factor of 1

Evaluation Criteria:

- | | |
|---|--------------|
| 1. Deflection at gears: | 127 mm |
| 2. Maximum angular deflection at bearings | < 0.004 rad |
| 3. Angular deflection between gear axes | < 0.0005 rad |
| 4. Angle of twist | < 3 deg/m |

Inputs From Engine

Maximum Power

$$P_{eng} := 24.4 \text{ kW} \quad \omega_{eng} := 6500 \text{ rpm} = 680.6784 \text{ Hz}$$

Primary Reduction

$$r_o := 2.272$$

Torque Calculation

Torque before reduction

$$T_o := \frac{P_{eng}}{\omega_{eng}} = 35.8466 \text{ N m}$$

Torque on input shaft

$$T_{in} := T_o \cdot r_o = 81.4435 \text{ N m}$$

☐ User Inputs

Shaft Properties: AISI 1080 Steel is used [1]

Ultimate Strength	$S_{ut} := 965 \text{ MPa}$	Shaft Length	$L := 425 \text{ mm}$
Yield Strength	$S_y := 585 \text{ MPa}$		
Young's Modulus	$E := 205 \text{ GPa}$	Design Factor of Safety:	$n_f := 1$
Shear Modulus	$G := 80.0 \text{ GPa}$	Operation Temperature (in Fahrenheit) [2]	$T_{oper} := 195$
Bore Diameter of shaft	$d_1 := 42 \text{ mm}$		
Diameter at bearings	$d_2 := 35 \text{ mm}$	Shoulder on shaft radius	$r := 3 \text{ mm}$

Gear Properties:

Pressure Angle

KAG Input Shaft Analysis

$\phi := 20 \text{ deg}$

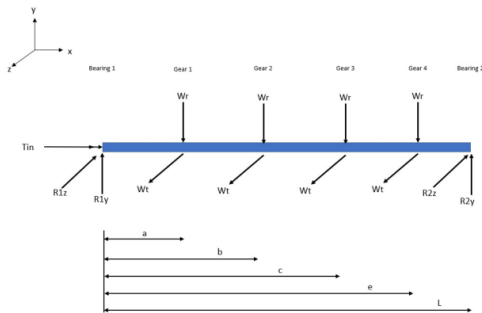
Gear Engaged	Pitch Diameter	Number of Teeth	Distance from left bearing
1	$d_{p1} := 355.6 \text{ mm}$	$N_1 := 42$	$a := 54.10 \text{ mm}$
2	$d_{p2} := 296.342 \text{ mm}$	$N_2 := 35$	$b := 174.40 \text{ mm}$
3	$d_{p3} := 237.058 \text{ mm}$	$N_3 := 28$	$c := 250.60 \text{ mm}$
4	$d_{p4} := 177.8 \text{ mm}$	$N_4 := 21$	$e := 370.90 \text{ mm}$

Gear Selection: Select 1,2,3, or 4

gear_pick := 1

Pitch Diameter	Number of Teeth	Distance from left bearing
if gear_pick = 1 $d_p := d_{p1}$	if gear_pick = 1 $N := N_1$	if gear_pick = 1 $L_{gear} := a$
else if gear_pick = 2 $d_p := d_{p2}$	else if gear_pick = 2 $N := N_2$	else if gear_pick = 2 $L_{gear} := b$
else if gear_pick = 3 $d_p := d_{p3}$	else if gear_pick = 3 $N := N_3$	else if gear_pick = 3 $L_{gear} := c$
else $d_p := d_{p4}$	else $N := N_4$	else $L_{gear} := e$

Forces from Gears and Support Reactions



Only the forces from one gear will be acting on the input shaft at any given time. The engaged gear selected by gear_pick will be analyzed.

Figure 1: Free Body Diagram (FBD) of the intermediate shaft and associate distances

$a := 54.10 \text{ mm}$ $b := 174.40 \text{ mm}$ $c := 250.60 \text{ mm}$ $e := 370.90 \text{ mm}$

Forces from gears:

Tangential Forces from Gears:

$$W_t := \frac{T_{in} \cdot 2}{d_p} = 458.0622 \text{ N}$$

Radial Forces from Gears:

$$W_r := W_t \cdot \tan(\phi) = 166.721 \text{ N}$$

KAG Input Shaft Analysis

Reaction Forces from support bearings:

$$R_{2y} := \frac{W_r \cdot L_{gear}}{L} = 21.2226 \text{ N}$$

$$R_{1y} := W_r - R_{2y} = 145.4984 \text{ N}$$

$$R_{2z} := \frac{W_t \cdot L_{gear}}{L} = 58.3086 \text{ N}$$

$$R_{1z} := W_t - R_{2z} = 399.7535 \text{ N}$$

□ Singularity Functions

$$S(x, a, n) := \begin{cases} (x - a)^n & \text{if } ((x - a) > 0) \wedge (n \geq 0) \\ 0 & \text{else} \end{cases}$$





Function	$q(x)$	Evaluation
Ramp	 $\langle x - a \rangle^{-1}$	$\begin{cases} 0, & \text{if } x < a \\ x - a, & \text{if } x \geq a \end{cases}$
Shear flow/ distributed load	 $\langle x - a \rangle^{-0}$	$\begin{cases} 0, & \text{if } x < a \\ 1, & \text{if } x \geq a \end{cases}$
Shear force/ support reactions	 $\langle x - a \rangle^{-1}$	$\begin{cases} 0, & \text{if } x \neq a \\ \pm\infty, & \text{if } x = a \end{cases}$
Moment/ couple (internal)	 $\langle x - a \rangle^{-2}$	$\begin{cases} 0, & \text{if } x \neq a \\ \pm\infty, & \text{if } x = a \end{cases}$

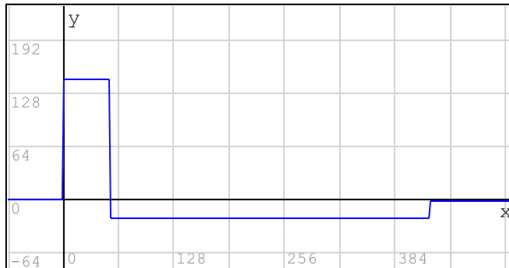
Figure 2: A guideline for finding singularity functions [3]

In x-y plane:

$$q_y(x) := R_{1y} \cdot S(x, 0, -1) - W_r \cdot S(x, L_{gear}, -1) + R_{2y} \cdot S(x, L, -1)$$

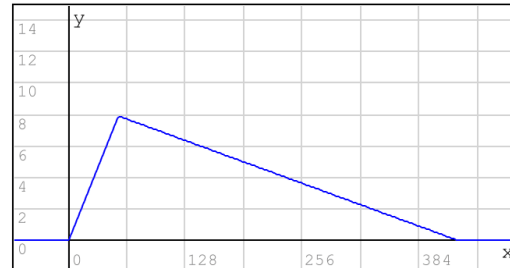
$$V_y(x) := R_{1y} \cdot S(x, 0, 0) - W_r \cdot S(x, L_{gear}, 0) + R_{2y} \cdot S(x, L, 0)$$

$$M_z(x) := R_{1y} \cdot S(x, 0, 1) - W_r \cdot S(x, L_{gear}, 1) + R_{2y} \cdot S(x, L, 1)$$



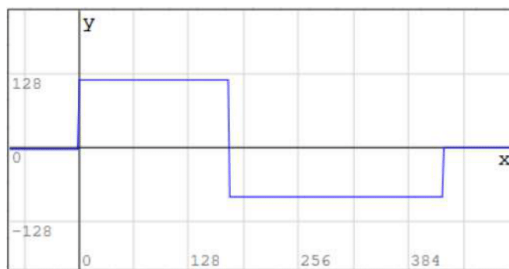
$V_y(x \text{ mm}) \text{ N}$

Figure 3: Shear Force Diagram (SFD) in the y-direction for gear 1



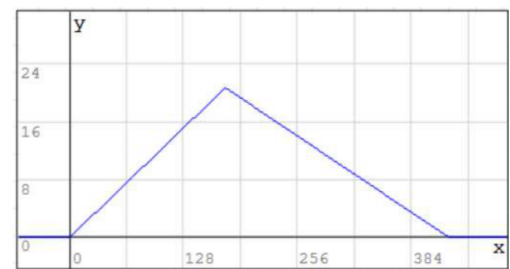
$M_z(x \text{ mm}) \text{ N m}$

Figure 4: Bending Moment Diagram (BMD) in the z-direction for gear 1



$V_y(x \text{ mm}) \text{ N}$

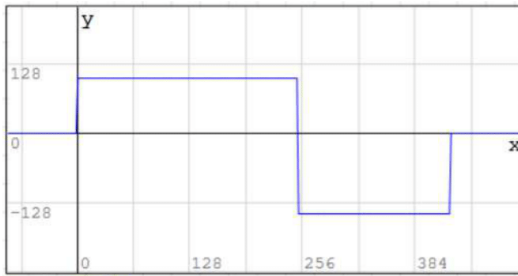
Figure 5: SFD in the y-direction for gear 2



$M_z(x \text{ mm}) \text{ N m}$

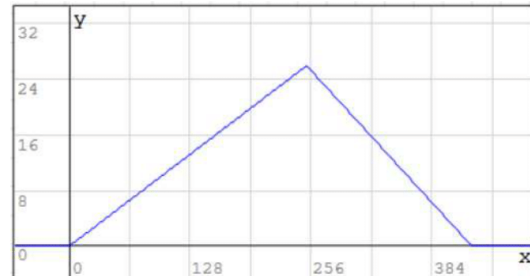
Figure 6: BMD in the z-direction for gear 2

KAG Input Shaft Analysis



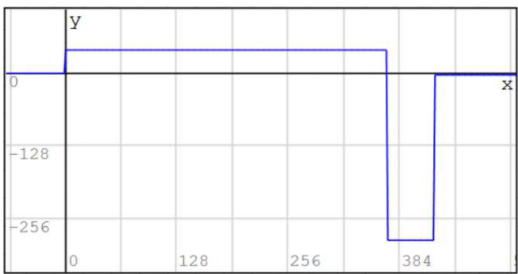
$V_y (x \text{ mm}) \text{ N}$

Figure 7: SFD in the y-direction for gear 3



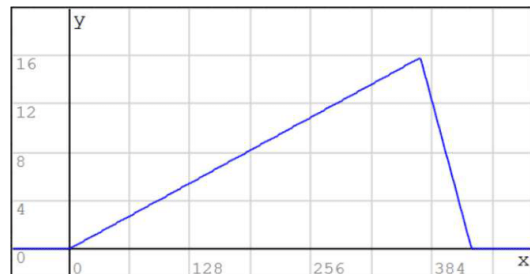
$M_z (x \text{ mm}) \text{ N m}$

Figure 8: BMD in the z-direction for gear 3



$V_y (x \text{ mm}) \text{ N}$

Figure 9: SFD in the y-direction for gear 4



$M_z (x \text{ mm}) \text{ N m}$

Figure 10: BMD in the z-direction for gear 4

In y-z plane:

$$q_z(x) := R_{1z} \cdot S(x, 0, -1) - W_t \cdot S(x, L_{gear}, -1) + R_{2z} \cdot S(x, L, -1)$$

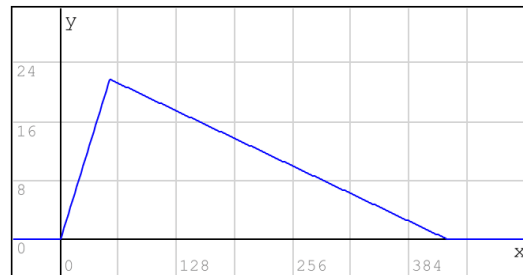
$$V_z(x) := R_{1z} \cdot S(x, 0, 0) - W_t \cdot S(x, L_{gear}, 0) + R_{2z} \cdot S(x, L, 0)$$

$$M_y(x) := R_{1z} \cdot S(x, 0, 1) - W_t \cdot S(x, L_{gear}, 1) + R_{2z} \cdot S(x, L, 1)$$



$V_z (x \text{ mm}) \text{ N}$

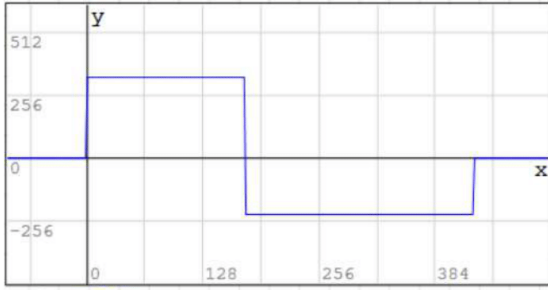
Figure 11: SFD in the z-direction for gear 1



$M_y (x \text{ mm}) \text{ N m}$

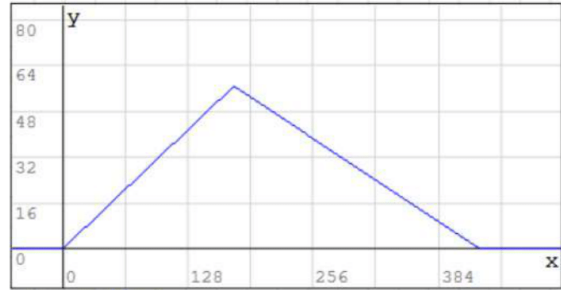
Figure 12: BMD in the y-direction for gear 1

KAG Input Shaft Analysis



$V_z (x \text{ mm}) \text{ N}$

Figure 13: SFD in the z-direction for gear 2



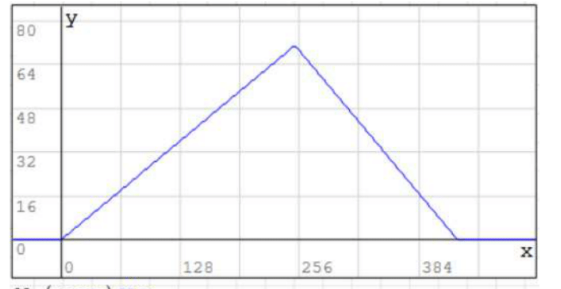
$M_y (x \text{ mm}) \text{ N m}$

Figure 14: BMD in the y-direction for gear 2



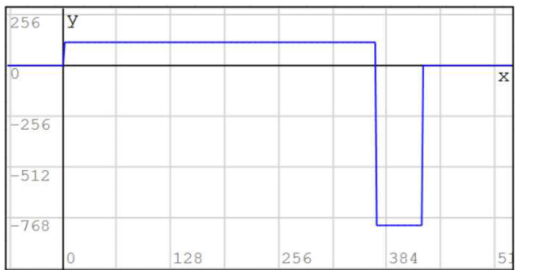
$V_z (x \text{ mm}) \text{ N}$

Figure 15: SFD in the z-direction for gear 3



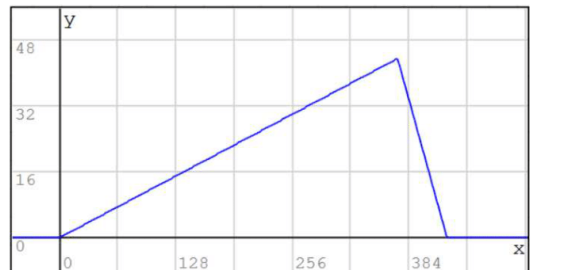
$M_y (x \text{ mm}) \text{ N m}$

Figure 16: BMD in the y-direction for gear 3



$V_z (x \text{ mm}) \text{ N}$

Figure 17: SFD in the z-direction for gear 4



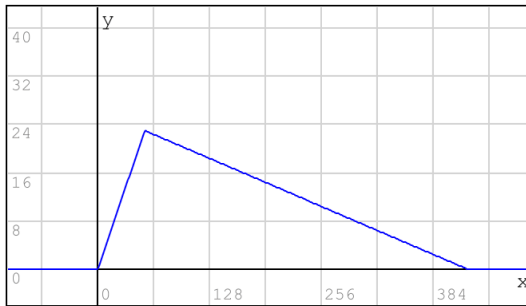
$M_y (x \text{ mm}) \text{ N m}$

Figure 18: BMD in the y-direction for gear 4

Sum of the Moment vectors:

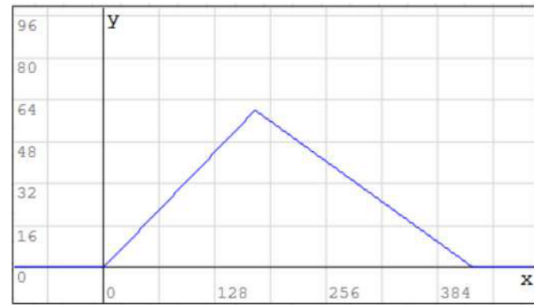
$$M(x) := \sqrt{(M_z(x))^2 + (M_y(x))^2}$$

KAG Input Shaft Analysis



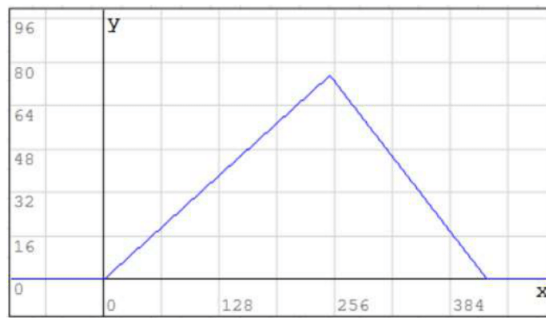
$M (x \text{ mm}) \text{ N m}$

Figure 19: Sum of the moment vectors for gear 1



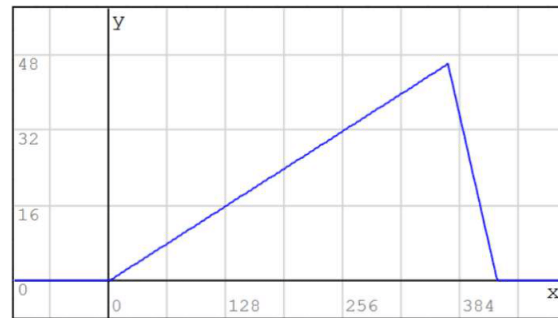
$M (x \text{ mm}) \text{ N m}$

Figure 20: Sum of the moment vectors for gear 2



$M (x \text{ mm}) \text{ N m}$

Figure 21: Sum of the moment vectors for gear 3



$M (x \text{ mm}) \text{ N m}$

Figure 22: Sum of the moment vectors for gear 4

Forces on Shaft from the gear:

Max moment will occur at the gear

$$M_{max} := M(L_{gear}) = 23.0146 \text{ N m}$$

$$M_{min} := -M_{max} = -23.0146 \text{ N m}$$

Alternating and Mean Moment

$$M_a := \left(\frac{M_{max} - M_{min}}{2} \right) = 23.0146 \text{ N m}$$

$$M_m := \frac{M_{max} + M_{min}}{2} = 0 \text{ N m}$$

Max torque is from the engine input

$$T_{max} := T_{in} = 81.4435 \text{ N m}$$

$$T_{min} := 0 \text{ N m}$$

Alternating and Mean Torque

$$T_a := \frac{T_{max} - T_{min}}{2} = 40.7217 \text{ N m}$$

$$T_m := \frac{T_{max} + T_{min}}{2} = 40.7217 \text{ N m}$$

□—Endurance Strength and Stress Concentration Factors

Uncorrected Endurance Strength:

if $S_{ut} \leq 1463 \text{ MPa}$

$$S'_e := 0.504 \cdot S_{ut}$$

else

$$S'_e := 737 \text{ MPa}$$

$$S'_e = 486.36 \text{ MPa}$$

KAG Input Shaft Analysis

Surface Condition Factor

Since it is assumed to be machined

$$k_a := 4.51 \cdot \left(\frac{S_{ut}}{\text{MPa}} \right)^{-0.265} = 0.7299$$

Surface finish	MPa		kpsi	
	a	b	a	B
Ground (standard unless otherwise indicated)	1.58	-0.085	1.34	-0.085
Machined or Cold drawn	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

Figure 23: Surface finish values [4]

Size Correction Factor

Calculated in while loop calculating minimum diameter

$$k_s = \begin{cases} 0.879d^{-0.157} & 0.11 \leq d \leq 2 \text{ in} \\ 0.9ld^{-0.157} & 2 < d \leq 10 \text{ in} \\ 1.24d^{-0.157} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.5ld^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

Figure 24: Size Correction Factor [4]

Load Correction Factor

Since bending is considered dominant

$$k_c := 1$$

$$k_c = \begin{cases} 1 & \text{Bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Pure torsion} \end{cases}$$

Figure 25: Load Correction Factor [4]

Temperature Correction Factor

$$k_d := 0.975 + 0.32 \cdot \left(10^{-3} \right) \cdot T_{oper} - 0.115 \cdot \left(10^{-5} \right) \cdot T_{oper}^2 + 0.104 \cdot \left(10^{-8} \right) \cdot T_{oper}^3 - 0.595 \cdot \left(10^{-12} \right) \cdot T_{oper}^4$$

$$k_d = 1.0005$$

Reliability Factor

Since reliability is assumed to be 90%

$$k_e := 0.897$$

•Base reliability 50%

Reliability	Ke
50	1
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.62

Figure 26: Reliability Factor [4]

Miscellaneous Correction Factor

$$k_f := 1$$

Endurance Strength:

Moment of Inertia

$$I_1 := \frac{\pi \cdot \left(d_1 \right)^4}{64} = 1.5275 \cdot 10^5 \text{ mm}^4$$

$$I_2 := \frac{\pi \cdot \left(d_2 \right)^4}{64} = 73661.7574 \text{ mm}^4$$

Nominal Alternating Bending Stress:

$$\sigma_{anom} := \frac{M_a \cdot \frac{d_2}{2}}{I_1} = 2.6368 \text{ MPa}$$

Nominal Alternating Torsional Stress:

Polar Moment of Inertia

$$J_1 := 2 \cdot I_1 = 3.0549 \cdot 10^5 \text{ mm}^4$$

$$J_2 := 2 \cdot I_2 = 1.4732 \cdot 10^5 \text{ mm}^4$$

Nominal Mean Bending Stress:

$$\sigma_{mnom} := \frac{M_m \cdot \frac{d_2}{2}}{I_1} = 0 \text{ MPa}$$

Nominal Mean Torsional Stress:

KAG Input Shaft Analysis

$$\tau_{anom} := \frac{T_a \cdot \frac{d_2}{2}}{J_1} = 2.3327 \text{ MPa}$$

$$\tau_{mnom} := \frac{T_m \cdot \frac{d_2}{2}}{J_1} = 2.3327 \text{ MPa}$$

2D Alternating Stress State (Von Mises):

2D Mean Stress State (Von Mises):

$$\sigma'_{anom} := \sqrt{\sigma_{anom}^2 + 3 \cdot \tau_{anom}^2} = 4.8247 \text{ MPa}$$

$$\sigma'_{mnom} := \sqrt{\sigma_{mnom}^2 + 3 \cdot \tau_{mnom}^2} = 4.0404 \text{ MPa}$$

$$\sigma'_{maxnom} := \sigma'_{anom} + \sigma'_{mnom} = 8.8651 \text{ MPa}$$

Finding K Values

Find k values from "Shaft with shoulder" charts

$$\frac{d_1}{d_2} = 1.2 \qquad \frac{r}{d_2} = 0.0857$$

$$K_{tstep} := 1.60$$

$$K_{tstep} := 1.24$$

$$q_{step} := 0.77$$

$$q_{step} := 0.81$$

K values from keyway

$$K_{tkey} := 2.2$$

$$K_{tskey} := 3$$

$$q_{key} := 0.78$$

Assuming the SCF for the keyway is greater:

$$K_t := K_{tkey} = 2.2$$

$$K_{ts} := K_{tskey} = 3$$

$$q := q_{key}$$

$$q_s := q_{key}$$

$$K_f := 1 + q \cdot (K_t - 1) = 1.936$$

$$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.56$$

$$K_{fm} := \text{if } K_f \cdot |\sigma'_{maxnom}| < S_y$$

$$\qquad K_f$$

$$\text{else}$$

$$\qquad \text{if } K_f \cdot |\sigma'_{maxnom}| > S_y$$

$$\qquad \frac{S_y - K_f \cdot \sigma'_{anom}}{\sigma'_{mnom}}$$

$$\text{else}$$

$$\qquad 0$$

$$K_{fms} := \text{if } K_f \cdot |\sigma'_{maxnom}| < S_y$$

$$\qquad K_{fs}$$

$$\text{else}$$

$$\qquad \text{if } K_f \cdot |\sigma'_{maxnom}| > S_y$$

$$\qquad \frac{S_y - K_{fs} \cdot \sigma'_{anom}}{\sigma'_{mnom}}$$

$$\text{else}$$

$$\qquad 0$$

$$K_{fm} = 1.936$$

$$K_{fms} = 2.56$$

Calculate Minimum Diameter

countIt := 0

$d_{min} := 0 \text{ mm}$

KAG Input Shaft Analysis

$d := 42 \text{ mm}$ Initial Guess

while $|d_{min} - d| > 0.00001 \text{ mm}$

```

    d := d_min
    k_b := if (d ≥ 2.79 mm) ∧ (d ≤ 51 mm)
            1.24 · (d/mm)-0.107
        else
            if (51 mm < d) ∧ (d ≤ 254 mm)
                1.51 · (d/mm)-0.157
            else
                if d > 254 mm
                    0.6
                else
                    1
            end if
        end if
    S_e := k_a · k_b · k_c · k_d · k_e · k_f · S'_e
    d_min := ⌊ (16 · n_f / π) · √ ( (4 · (K_f · M_a)2 + 3 · (K_fs · T_a)2) / S_e2 + (4 · (K_f · M_m)2 + 3 · (K_fs · T_m)2) / S_y2 ) ⌋1/3
    countIt := countIt + 1

```

countIt = 5

$d_{min} = 15.6347 \text{ mm}$

```

d_check := if d_min < d_1
            "Diameter of 42 mm is large enough"
        else
            "Diameter is too small"
        end if

```

☐—Verify using Evaluation Criteria

Slope and Deflection Calculations:

In y-direction

$$\theta_z(x) := \frac{1}{E} \cdot \left(\frac{R_{1y}}{2 \cdot I_2} \cdot S(x, 0, 2) - \frac{W_r}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2y}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1y}$$

$$y(x) := \frac{1}{E} \cdot \left(\frac{R_{1y}}{6 \cdot I_2} \cdot S(x, 0, 3) - \frac{W_r}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2y}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1y} \cdot x + C_{2y}$$

Solving for constants:

$$y(0 \text{ mm}) = 0 \text{ mm} \quad y(L) = 0 \text{ mm}$$

$$C_{2y} := 0$$

KAG Input Shaft Analysis

$$C_{1y} := \frac{\left(-\frac{1}{E} \cdot \left(\frac{R_{1y}}{6 \cdot I_2} \cdot S(L, 0, 3) - \frac{W_r}{6 \cdot I_1} \cdot S(L, L_{gear}, 3) + \frac{R_{2y}}{6 \cdot I_2} \cdot S(L, L, 3) \right) + C_{2y} \right)}{L} = -0.0002$$

$$\theta_z(x) := \frac{1}{E} \cdot \left(\frac{R_{1y}}{2 \cdot I_2} \cdot S(x, 0, 2) - \frac{W_r}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2y}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1y}$$

$$y(x) := \frac{1}{E} \cdot \left(\frac{R_{1y}}{6 \cdot I_2} \cdot S(x, 0, 3) - \frac{W_r}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2y}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1y} \cdot x + C_{2y}$$

In z-direction:

$$\theta_y(x) := \frac{1}{E} \cdot \left(\frac{R_{1z}}{2 \cdot I_2} \cdot S(x, 0, 2) - \frac{W_t}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2z}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1z}$$

$$z(x) := \frac{1}{E} \cdot \left(\frac{R_{1z}}{6 \cdot I_2} \cdot S(x, 0, 3) - \frac{W_t}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2z}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1z} \cdot x + C_{2z}$$

Solving for constants:

$$z(0 \text{ mm}) = 0 \text{ mm} \quad z(L) = 0 \text{ mm}$$

$$C_{2z} := 0$$

$$C_{1z} := \frac{\left(-\frac{1}{E} \cdot \left(\frac{R_{1z}}{6 \cdot I_2} \cdot S(L, 0, 3) - \frac{W_t}{6 \cdot I_1} \cdot S(L, L_{gear}, 3) + \frac{R_{2z}}{6 \cdot I_2} \cdot S(L, L, 3) \right) + C_{2z} \right)}{L} = -0.0005$$

$$\theta_y(x) := \frac{1}{E} \cdot \left(\frac{R_{1z}}{2 \cdot I_2} \cdot S(x, 0, 2) - \frac{W_t}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2z}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1z}$$

$$z(x) := \frac{1}{E} \cdot \left(\frac{R_{1z}}{6 \cdot I_2} \cdot S(x, 0, 3) - \frac{W_t}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2z}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1z} \cdot x + C_{2z}$$

Total Angular Deflection

$$\theta(x) := \sqrt{(\theta_y(x))^2 + (\theta_z(x))^2}$$

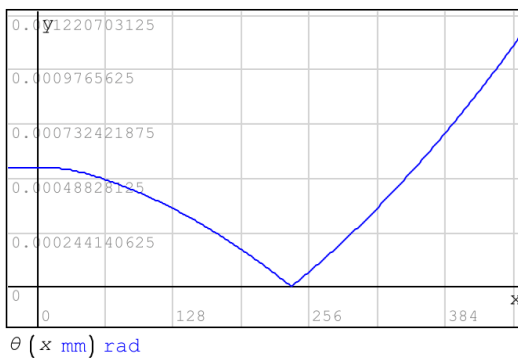


Figure 27: Total angular deflection for gear 1

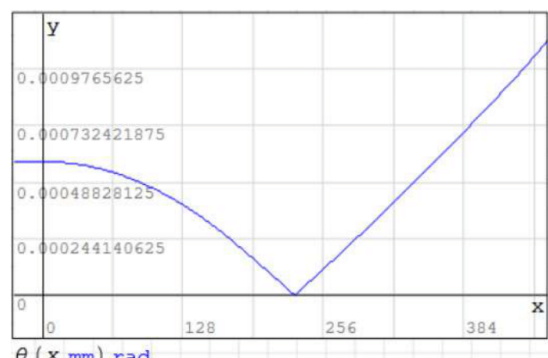


Figure 28: Total angular deflection for gear 2

KAG Input Shaft Analysis

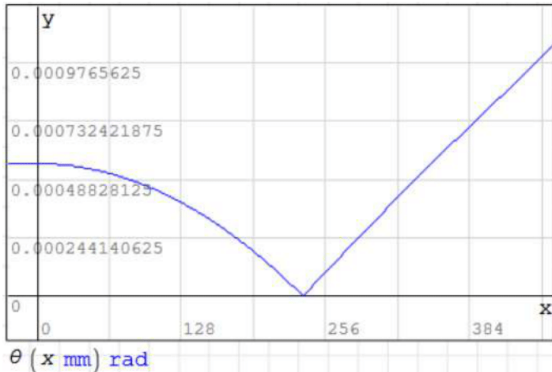


Figure 29: Total angular deflection for gear 3

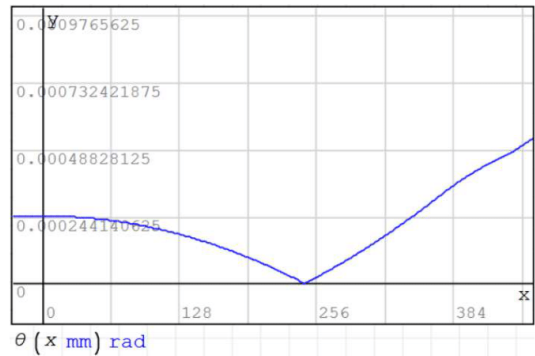


Figure 30: Total angular deflection for gear 4

Total Deflection

$$\delta (x) := \sqrt{z(x)^2 + y(x)^2}$$

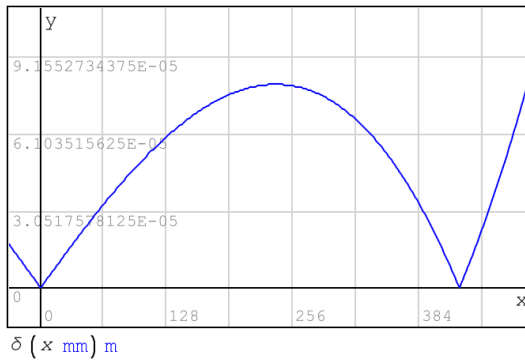


Figure 31: Total deflection for gear 1

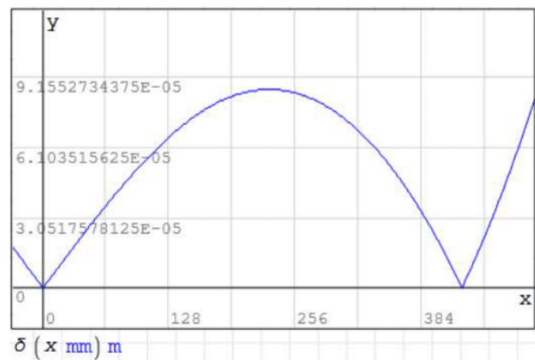


Figure 32: Total deflection for gear 2

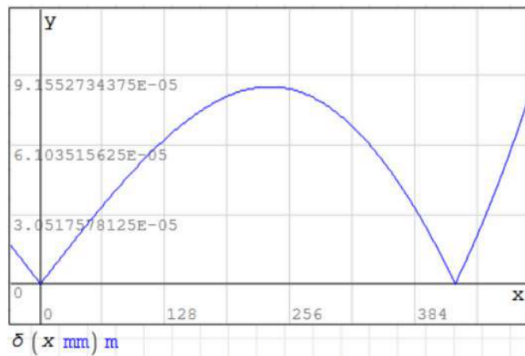


Figure 33: Total deflection for gear 3

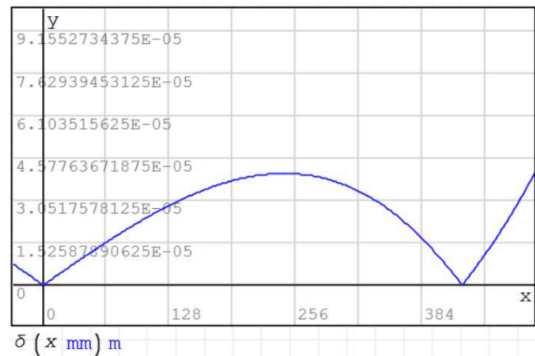


Figure 34: Total deflection for gear 4

Compare to evaluation criteria:

Deflection at the gear:

$$g_deflection := \delta (L_{gear}) = 0.0283 \text{ mm}$$

KAG Input Shaft Analysis

```
deflec_verify := if (g_deflection > 127 mm)
    "There is too much deflection at the gear "
else
    "Deflection at the gears was determined to be acceptable"
```

Angular Deflection at the gear:

$$g_angdeflec := \theta (L_{gear}) = 0.0005 \text{ rad}$$

```
g_angdeflec_verify := if (g_angdeflec > 0.0005 rad)
    "There is too much angular deflection at the gear"
else
    "Angular deflection at the gears was determined to be acceptable"
```

Angular Deflection at the bearings:

$$bear1_angdeflec := \theta (0 \text{ mm}) = 0.0005 \text{ rad}$$

$$bear2_angdeflec := \theta (L) = 0.0009 \text{ rad}$$

```
bear_angdeflec_verify := if (bear1_angdeflec > 0.004 rad)
    "There was determined to be too much angular deflection at bearing 1"
else
    if (bear2_angdeflec > 0.004 rad)
        "There was determined to be too much angular deflection at bearing 2"
    else
        "Angular deflection at the bearings was determined to be acceptable"
```

Angle of Twist:

$$\phi := \frac{T_{in}}{J_1 \cdot G} = 0.1909377 \frac{\text{deg}}{\text{m}}$$

```
angletwist_verify := if (phi > 3 deg/m)
    "The angle of twist was determined to be too large"
else
    "The angle of twist was determined to be acceptable"
```

deflec_verify = "Deflection at the gears was determined to be acceptable"

bear_angdeflec_verify = "Angular deflection at the bearings was determined to be acceptable"

g_angdeflec_verify = "Angular deflection at the gears was determined to be acceptable"

angletwist_verify = "The angle of twist was determined to be acceptable"

d_check = "Diameter of 42 mm is large enough"

Alternating Bending Stress:

$$\sigma_a := \frac{K_f \cdot M_a \cdot \frac{d_1}{2}}{I_1} = 6.1258 \text{ MPa}$$

Mean Bending Stress:

$$\sigma_m := \frac{K_f \cdot M_m \cdot \frac{d_1}{2}}{I_1} = 0 \text{ MPa}$$

Alternating Torsional Stress:

$$\tau_a := \frac{K_{fs} \cdot T_a \cdot \frac{d_1}{2}}{J_1} = 7.1662 \text{ MPa}$$

Mean Torsional Stress:

$$\tau_m := \frac{K_{fs} \cdot T_m \cdot \frac{d_1}{2}}{J_1} = 7.1662 \text{ MPa}$$

2D Alternating Stress State (Von Mises):

2D Mean Stress State (Von Mises):

KAG Input Shaft Analysis

$$\sigma'_a := \sqrt{\sigma_a^2 + 3 \cdot \tau_a^2} = 13.8415 \text{ MPa}$$

$$\sigma'_m := \sqrt{\sigma_m^2 + 3 \cdot \tau_m^2} = 12.4122 \text{ MPa}$$

Calculated Safety Factors:

$$n_f := \frac{1}{\sqrt{\left(\frac{\sigma'_a}{S_e}\right)^2 + \left(\frac{\sigma'_m}{S_y}\right)^2}} = 19.3855$$

Fatigue Safety Factor

$$n_y := \frac{1}{\frac{\sigma'_a}{S_y} + \frac{\sigma'_m}{S_y}} = 22.2825$$

Yield Safety Factor

— Summarizing Minimum Diameters and verifying if Evaluation Criteria was met

Summary of values found

Gear	Minimum Diameter (mm)	Deflection at gear (mm)	Ang. Deflec. at gear (rad)	Max Ang. Deflec. at bearing (rad)	Angle of Twist (deg/m)
1	15.6347	0.0283	0.0005	0.0009	0.1909377
2	17.5394	0.0796	0.0002	0.0009	0.1909377
3	18.4034	0.0856	$7.4329 \cdot 10^{-5}$	0.0009	0.1909377
4	16.742	0.0212	0.0003	0.0005	0.1909377

Safety Factors

Gear	Fatigue Safety Factor	Yield Safety Factor
1	19.3855	22.2825
2	13.731	17.9061
3	11.8866	16.2667
4	15.7881	19.5964

Summary of whether or not evaluation criteria was met

Gear	Minimum Diameter	Deflection at gear	Ang. Deflec. at gear	Max Ang. Deflec. at bearing	Angle of Twist
1	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable
2	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable
3	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable
4	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable

KAG Output Shaft Analysis

KAG Output Shaft Analysis

☐—Assumptions, Evaluation Criteria, Torque Calculations

Assumptions:

1. Gears used are spur gears
2. Shafts are designed with 90% reliability and infinite life, with no miscellaneous factors
3. Shafts are machined
4. Bending is dominant on both shafts
5. Torque application is constant
6. The shaft is supported by two bearings at the ends
7. The bearing and gear forces act as point loads
8. All dimensions and sizes fit within the normal range of values
9. There is no undercutting or interference of the gears
10. There is a safety factor of 1

Evaluation Criteria:

- | | |
|---|--------------|
| 1. Deflection at gears: | 127 mm |
| 2. Maximum angular deflection at bearings | < 0.004 rad |
| 3. Angular deflection between gear axes | < 0.0005 rad |
| 4. Angle of twist | < 3 deg/m |

Inputs From Engine

Maximum Power

$$P_{eng} := 24.4 \text{ kW} \quad \omega_{eng} := 6500 \text{ rpm} = 680.6784 \text{ Hz}$$

Primary Reduction

$$r_0 := 2.272$$

Torque Calculation

Torque before reduction

$$T_0 := \frac{P_{eng}}{\omega_{eng}} = 35.8466 \text{ N m}$$

Torque on input shaft

$$T_{in} := T_0 \cdot r_0 = 81.4435 \text{ N m}$$

☐—User Inputs

Shaft Properties: AISI 1080 Steel is used [1]

Ultimate Strength	$S_{ut} := 965 \text{ MPa}$	Shaft Length	$L := 425 \text{ mm}$
Yield Strength	$S_y := 585 \text{ MPa}$		
Young's Modulus	$E := 205 \text{ GPa}$	Design Factor of Safety:	$n_f := 1$
Shear Modulus	$G := 80.0 \text{ GPa}$	Operation Temperature (in Fahrenheit) [2]	$T_{oper} := 195$
Bore Diameter of shaft	$d_1 := 43 \text{ mm}$		
Diameter at bearings	$d_2 := 40 \text{ mm}$	Shoulder on shaft radius	$r := 3 \text{ mm}$

Gear Properties:

Pressure Angle

KAG Output Shaft Analysis

$$\phi := 20 \text{ deg}$$

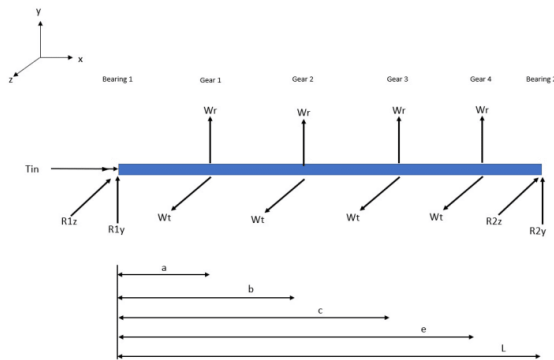
Gear Engaged	Pitch Diameter	Number of Teeth	Distance from left bearing	Gear Reduction
1	$d_{p1} := 177.8 \text{ mm}$	$N_1 := 21$	$a := 54.10 \text{ mm}$	$r_1 := \frac{42}{21} = 2$
2	$d_{p2} := 237.058 \text{ mm}$	$N_2 := 28$	$b := 174.40 \text{ mm}$	$r_2 := \frac{35}{28} = 1.25$
3	$d_{p3} := 296.342 \text{ mm}$	$N_3 := 35$	$c := 250.60 \text{ mm}$	$r_3 := \frac{28}{35} = 0.8$
4	$d_{p4} := 355.6 \text{ mm}$	$N_4 := 42$	$e := 370.90 \text{ mm}$	$r_4 := \frac{21}{42} = 0.5$

Gear Selection: Select 1,2,3, or 4

$$\text{gear_pick} := 1$$

Pitch Diameter	Number of Teeth	Distance from left bearing	Torque from Gear Reduction
if gear_pick = 1 $d_p := d_{p1}$	if gear_pick = 1 $N := N_1$	if gear_pick = 1 $L_{gear} := a$	if gear_pick = 1 $T := T_{in} \cdot r_1$
else if gear_pick = 2 $d_p := d_{p2}$	else if gear_pick = 2 $N := N_2$	else if gear_pick = 2 $L_{gear} := b$	else if gear_pick = 2 $T := T_{in} \cdot r_2$
else if gear_pick = 3 $d_p := d_{p3}$	else if gear_pick = 3 $N := N_3$	else if gear_pick = 3 $L_{gear} := c$	else if gear_pick = 3 $T := T_{in} \cdot r_3$
else $d_p := d_{p4}$	else $N := N_4$	else $L_{gear} := e$	else $T := T_{in} \cdot r_4$

Force Analysis of Gears and Support Reactions



Only the forces from one gear will be acting on the output shaft at any given time

Figure 1: Free Body Diagram (FBD) of the intermediate shaft and associated distances

$$a := 54.10 \text{ mm} \quad b := 174.40 \text{ mm} \quad c := 250.60 \text{ mm} \quad e := 370.90 \text{ mm}$$

Forces from gears:

Tangential Forces from Gears:

$$W_t := \frac{T_{in} \cdot 2}{d_p} = 916.1243 \text{ N}$$

Radial Forces from Gears:

$$W_r := W_t \cdot \tan(\phi) = 333.442 \text{ N}$$

Reaction Forces from support bearings:

KAG Output Shaft Analysis

$$R_{2y} := \frac{(-W_r) \cdot L_{gear}}{L} = -42.4452 \text{ N} \quad R_{1y} := -W_r - R_{2y} = -290.9968 \text{ N}$$

$$R_{2z} := \frac{(-W_t) \cdot L_{gear}}{L} = -116.6172 \text{ N} \quad R_{1z} := -W_t - R_{2z} = -799.5071 \text{ N}$$

□ Singularity Functions

$$S(x, a, n) := \text{if} \left((x - a) > 0 \right) \wedge (n \geq 0) \begin{cases} (x - a)^n \\ \text{else} \\ 0 \end{cases}$$

Function	$q(x)$	Evaluation
Ramp		$\langle x - a \rangle^{-1} = \begin{cases} 0, & \text{if } x < a \\ x - a, & \text{if } x \geq a \end{cases}$
Shear flow/ distributed load		$\langle x - a \rangle^{-0} = \begin{cases} 0, & \text{if } x < a \\ 1, & \text{if } x \geq a \end{cases}$
Shear force/ support reactions		$\langle x - a \rangle^{-1} = \begin{cases} 0, & \text{if } x \neq a \\ +\infty, & \text{if } x = a \end{cases}$
Moment/ couple (internal)		$\langle x - a \rangle^{-2} = \begin{cases} 0, & \text{if } x \neq a \\ \pm\infty, & \text{if } x = a \end{cases}$

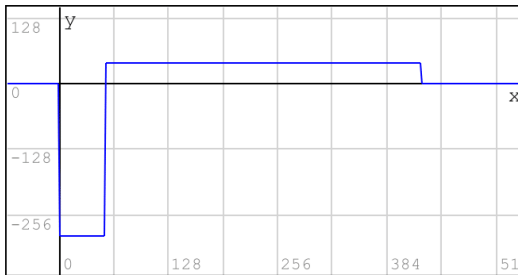
Figure 2: A guideline for finding singularity functions [3]

In x-y plane:

$$q_y(x) := R_{1y} \cdot S(x, 0, -1) + W_r \cdot S(x, L_{gear}, -1) + R_{2y} \cdot S(x, L, -1)$$

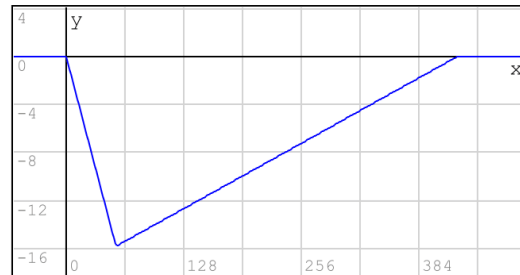
$$V_y(x) := R_{1y} \cdot S(x, 0, 0) + W_r \cdot S(x, L_{gear}, 0) + R_{2y} \cdot S(x, L, 0)$$

$$M_z(x) := R_{1y} \cdot S(x, 0, 1) + W_r \cdot S(x, L_{gear}, 1) + R_{2y} \cdot S(x, L, 1)$$



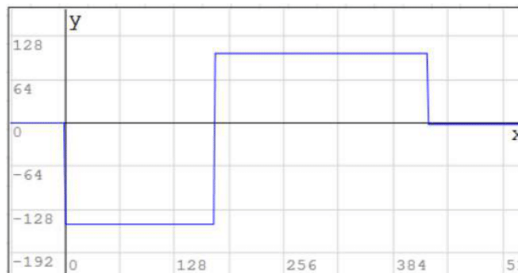
$V_y(x \text{ mm}) \text{ N}$

Figure 3: Shear Force Diagram (SFD) in the y-direction for gear 1



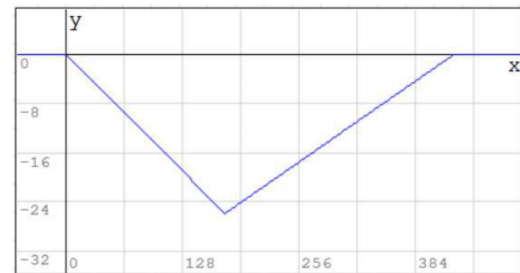
$M_z(x \text{ mm}) \text{ N m}$

Figure 4: Bending Moment Diagram (BMD) in the z-direction for gear 1



$V_y(x \text{ mm}) \text{ N}$

Figure 5: SFD in the y-direction for gear 2



$M_z(x \text{ mm}) \text{ N m}$

Figure 6: BMD in the z-direction for gear 2

KAG Output Shaft Analysis



Figure 7: SFD in the y-direction for gear 3

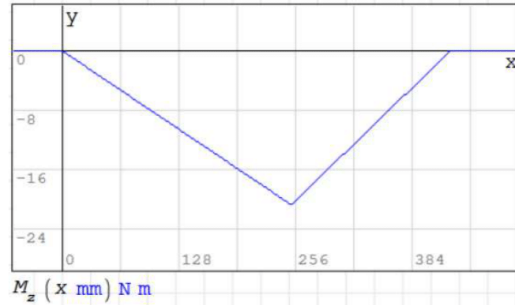


Figure 8: BMD in the z-direction for gear 3

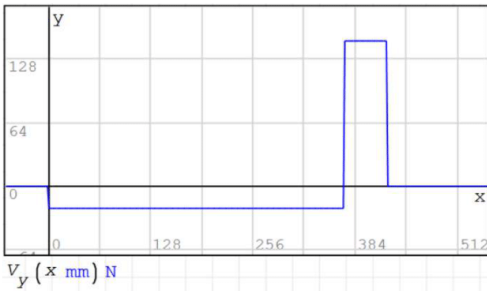


Figure 9: SFD in the y-direction for gear 4

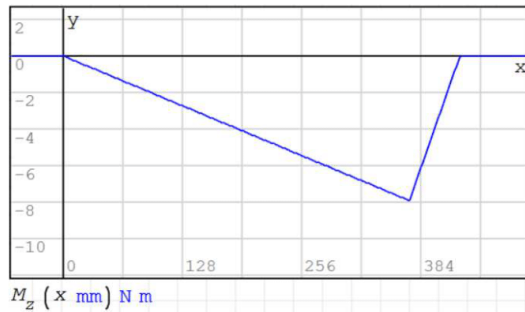


Figure 10: BMD in the z-direction for gear 4

In y-z plane:

$$q_z(x) := R_{1z} \cdot S(x, 0, -1) + W_t \cdot S(x, L_{gear}, -1) + R_{2z} \cdot S(x, L, -1)$$

$$V_z(x) := R_{1z} \cdot S(x, 0, 0) + W_t \cdot S(x, L_{gear}, 0) + R_{2z} \cdot S(x, L, 0)$$

$$M_y(x) := R_{1z} \cdot S(x, 0, 1) + W_t \cdot S(x, L_{gear}, 1) + R_{2z} \cdot S(x, L, 1)$$

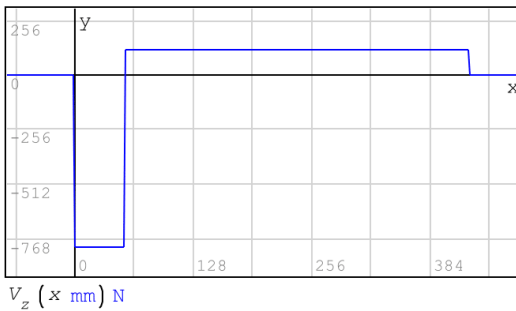


Figure 11: SFD in the z-direction for gear 1

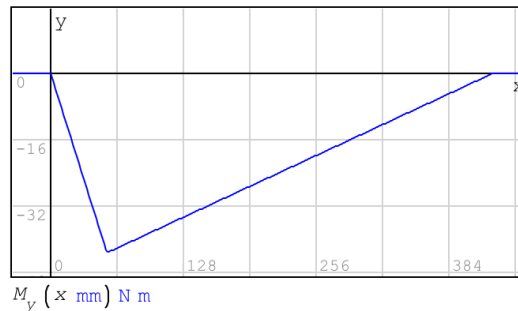


Figure 12: BMD in the y-direction for gear 1

KAG Output Shaft Analysis

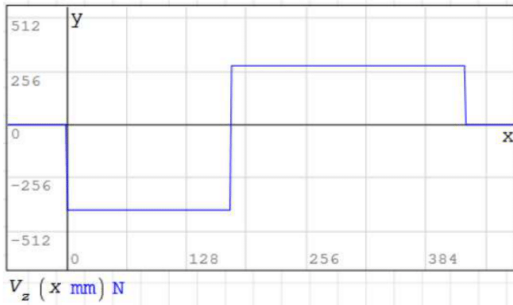


Figure 13: SFD in the z-direction for gear 2

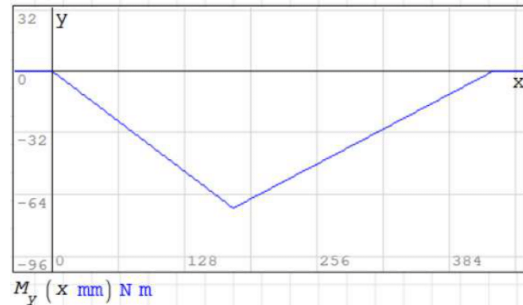


Figure 14: BMD in the y-direction for gear 2

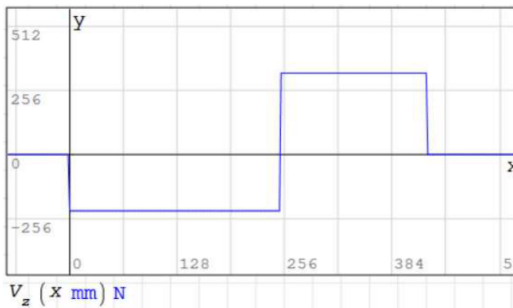


Figure 15: SFD in the z-direction for gear 3

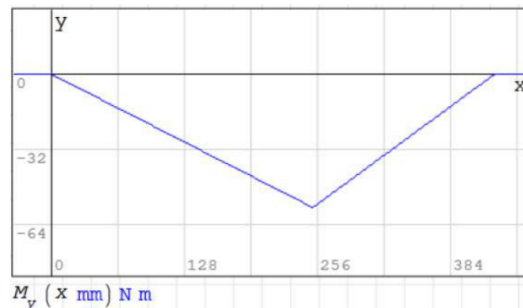


Figure 16: BMD in the y-direction for gear 3

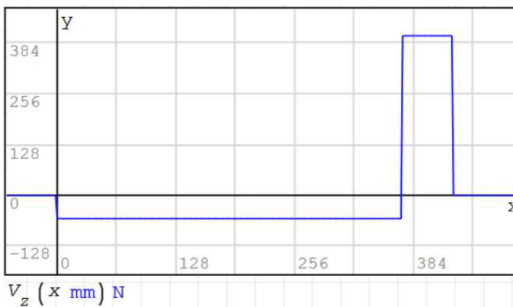


Figure 17: SFD in the z-direction for gear 4

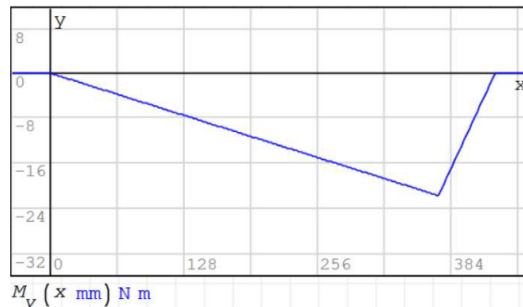


Figure 18: BMD in the y-direction for gear 4

Sum of the Moment vectors:

$$M(x) := \sqrt{(M_z(x))^2 + (M_y(x))^2}$$

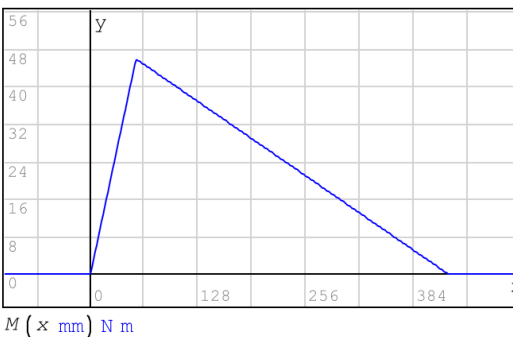


Figure 19: Sum of the moment vectors for gear 1

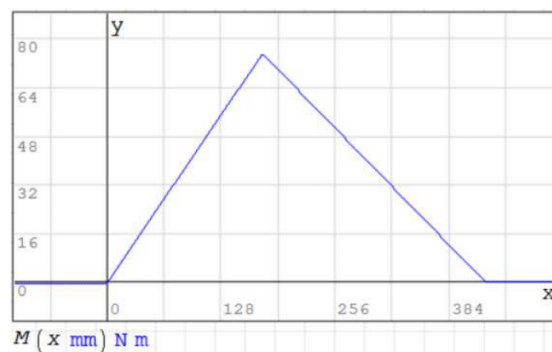


Figure 20: Sum of the moment vectors for gear 2

KAG Output Shaft Analysis

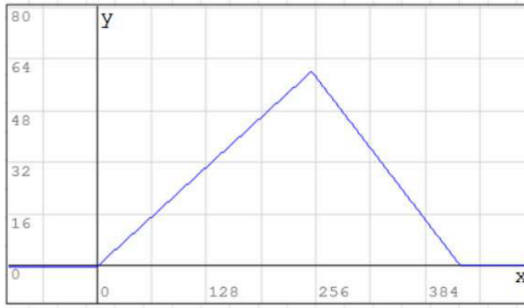


Figure 21: Sum of the moment vectors for gear 3

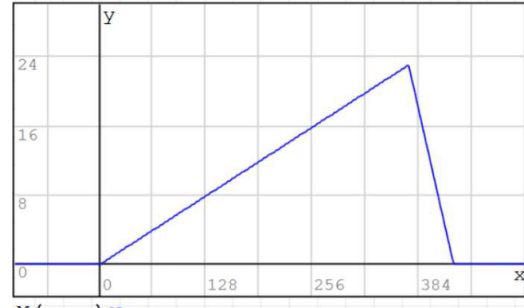


Figure 22: Sum of the moment vectors for gear 4

Forces on Shaft from the gear:

Max moment will occur at the gear

$$M_{max} := M(L_{gear}) = 46.0292 \text{ N m}$$

$$M_{min} := -M_{max} = -46.0292 \text{ N m}$$

Alternating and Mean Moment

$$M_a := \left(\frac{M_{max} - M_{min}}{2} \right) = 46.0292 \text{ N m}$$

$$M_m := \frac{M_{max} + M_{min}}{2} = 0 \text{ N m}$$

Max torque is from the engine input

$$T_{max} := T_{in} = 81.4435 \text{ N m}$$

$$T_{min} := 0 \text{ N m}$$

Alternating and Mean Torque

$$T_a := \frac{T_{max} - T_{min}}{2} = 40.7217 \text{ N m}$$

$$T_m := \frac{T_{max} + T_{min}}{2} = 40.7217 \text{ N m}$$

Endurance Strength and Stress Concentration Factors

Uncorrected Endurance Strength:

if $S_{ut} \leq 1463 \text{ MPa}$

$$S'_e := 0.504 \cdot S_{ut}$$

else

$$S'_e := 737 \text{ MPa}$$

$$S'_e = 486.36 \text{ MPa}$$

Surface Condition Factor

Since it is assumed to be machined

$$k_a := 4.51 \cdot \left(\frac{S_{ut}}{\text{MPa}} \right)^{-0.265} = 0.7299$$

Surface finish	MPa		kpsi	
	a	b	a	B
Ground (standard unless otherwise indicated)	1.58	-0.085	1.34	-0.085
Machined or Cold drawn	4.51	-0.265	2.7	-0.265
Hot-rolled	57.7	-0.718	14.4	-0.718
As-forged	272	-0.995	39.9	-0.995

Figure 23: Surface finish values [4]

Size Correction Factor

Calculated in while loop calculating minimum diameter

$$k_s = \begin{cases} 0.879d^{-0.187} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ 1.24d^{-0.187} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

Figure 24: Size Correction Factor [4]

Load Correction Factor

KAG Output Shaft Analysis

Since bending is considered dominant

$$k_c := 1$$

$$k_c = \begin{cases} 1 & \text{Bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Pure torsion} \end{cases}$$

Figure 25: Load Correction Factor [4]

Temperature Correction Factor

$$k_d := 0.975 + 0.32 \cdot (10^{-3}) \cdot T_{oper} - 0.115 \cdot (10^{-5}) \cdot T_{oper}^2 + 0.104 \cdot (10^{-8}) \cdot T_{oper}^3 - 0.595 \cdot (10^{-12}) \cdot T_{oper}^4$$

$$k_d = 1.0005$$

Reliability Factor

Since reliability is assumed to be 90%

$$k_e := 0.897$$

•Base reliability 50%

Reliability	Ke
50	1
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.62

Figure 26: Reliability Factor [4]

Miscellaneous Correction Factor

$$k_f := 1$$

Endurance Strength:

Moment of Inertia

$$I_1 := \frac{\pi \cdot (d_1^4)}{64} = 1.6782 \cdot 10^5 \text{ mm}^4$$

$$I_2 := \frac{\pi \cdot (d_2^4)}{64} = 1.2566 \cdot 10^5 \text{ mm}^4$$

Polar Moment of Inertia

$$J_1 := 2 \cdot I_1 = 3.3564 \cdot 10^5 \text{ mm}^4$$

$$J_2 := 2 \cdot I_2 = 2.5133 \cdot 10^5 \text{ mm}^4$$

Nominal Alternating Bending Stress:

$$\sigma_{anom} := \frac{M_a \cdot \frac{d_1}{2}}{I_1} = 5.897 \text{ MPa}$$

Nominal Mean Bending Stress:

$$\sigma_{mnom} := \frac{M_m \cdot \frac{d_1}{2}}{I_1} = 0 \text{ MPa}$$

Nominal Alternating Torsional Stress:

$$\tau_{anom} := \frac{T_a \cdot \frac{d_1}{2}}{J_1} = 2.6085 \text{ MPa}$$

Nominal Mean Torsional Stress:

$$\tau_{mnom} := \frac{T_m \cdot \frac{d_1}{2}}{J_1} = 2.6085 \text{ MPa}$$

2D Alternating Stress State (Von Mises):

$$\sigma'_{anom} := \sqrt{\sigma_{anom}^2 + 3 \cdot \tau_{anom}^2} = 7.4288 \text{ MPa}$$

2D Mean Stress State (Von Mises):

$$\sigma'_{mnom} := \sqrt{\sigma_{mnom}^2 + 3 \cdot \tau_{mnom}^2} = 4.5181 \text{ MPa}$$

$$\sigma'_{maxnom} := \sigma'_{anom} + \sigma'_{mnom} = 11.9469 \text{ MPa}$$

Finding K Values

Find k values from "Shaft with shoulder" charts

KAG Output Shaft Analysis

$$\frac{d_1}{d_2} = 1.075 \quad \frac{r}{d_2} = 0.075$$

$$K_{tstep} := 1.60$$

$$K_{tsstep} := 1.24$$

$$q_{step} := 0.77$$

$$q_{sstep} := 0.81$$

K values from keyway

$$K_{tkey} := 2.2$$

$$K_{tskey} := 3$$

$$q_{key} := 0.78$$

Assuming the SCF for the keyway is greater:

$$K_t := K_{tkey} = 2.2$$

$$K_{ts} := K_{tskey} = 3$$

$$q := q_{key}$$

$$q_s := q_{key}$$

$$K_f := 1 + q \cdot (K_t - 1) = 1.936$$

$$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.56$$

$$K_{fm} := \begin{cases} K_f \cdot \left| \sigma'_{maxnom} \right| < S_y & K_f \\ \text{else} & \text{if } K_f \cdot \left| \sigma'_{maxnom} \right| > S_y \\ & \frac{S_y - K_f \cdot \sigma'_{anom}}{\sigma'_{mnom}} \\ \text{else} & 0 \end{cases}$$

$$K_{fms} := \begin{cases} K_{fs} \cdot \left| \sigma'_{maxnom} \right| < S_y & K_{fs} \\ \text{else} & \text{if } K_{fs} \cdot \left| \sigma'_{maxnom} \right| > S_y \\ & \frac{S_y - K_{fs} \cdot \sigma'_{anom}}{\sigma'_{mnom}} \\ \text{else} & 0 \end{cases}$$

$$K_{fm} = 1.936$$

$$K_{fms} = 2.56$$

☐ Calculate Minimum Diameter

countIt := 0

$d_{min} := 0$ mm

$d := 43$ mm Initial Guess

KAG Output Shaft Analysis

```

while |d_min - d| > 0.00001 mm
  d := d_min
  k_b := if (d ≥ 2.79 mm) ∧ (d ≤ 51 mm)
    1.24 · (d/mm)^-0.107
  else
    if (51 mm < d) ∧ (d ≤ 254 mm)
      1.51 · (d/mm)^-0.157
    else
      if d > 254 mm
        0.6
      else
        1
  S_e := k_a · k_b · k_c · k_d · k_e · k_f · S'_e
  d_min := (16 · n_f / π) · √( (4 · (K_f · M_a)^2 + 3 · (K_fs · T_a)^2) / S_e^2 + (4 · (K_f · M_m)^2 + 3 · (K_fs · T_m)^2) / S_y^2 )^(1/3)
  countIt := countIt + 1

countIt = 6
d_min = 16.742 mm
d_check := if d_min < d_1
  "Diameter of 42 mm is large enough"
else
  "Diameter is too small"

```

☐—Verify using Evaluation Criteria

Slope and Deflection Calculations:

In y-direction

$$\theta_z(x) := \frac{1}{E} \cdot \left[\frac{R_{1y}}{2 \cdot I_2} \cdot S(x, 0, 2) + \frac{W_r}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2y}}{2 \cdot I_2} \cdot S(x, L, 2) \right] + C_{1y}$$

$$y(x) := \frac{1}{E \cdot I} \cdot \left[\frac{R_{1y}}{6 \cdot I_2} \cdot S(x, 0, 3) + \frac{W_r}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2y}}{6 \cdot I_2} \cdot S(x, L, 3) \right] + C_{1y} \cdot x + C_{2y}$$

Solving for constants:

$$y(0 \text{ mm}) = 0 \text{ mm} \quad y(L) = 0 \text{ mm}$$

$$C_{2y} := 0$$

$$C_{1y} := \frac{\left(-\frac{1}{E} \right) \cdot \left[\frac{R_{1y}}{6 \cdot I_2} \cdot S(L, 0, 3) + \frac{W_r}{6 \cdot I_1} \cdot S(L, L_{gear}, 3) + \frac{R_{2y}}{6 \cdot I_2} \cdot S(L, L, 3) \right] + C_{2y}}{L} = 0.0001$$

KAG Output Shaft Analysis

$$\theta_z(x) := \frac{1}{E} \cdot \left(\frac{R_{1y}}{2 \cdot I_2} \cdot S(x, 0, 2) + \frac{W_r}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2y}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1y}$$

$$y(x) := \frac{1}{E} \cdot \left(\frac{R_{1y}}{6 \cdot I_2} \cdot S(x, 0, 3) + \frac{W_r}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2y}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1y} \cdot x + C_{2y}$$

In z-direction:

$$\theta_y(x) := \frac{1}{E \cdot I} \cdot \left(\frac{R_{1z}}{2 \cdot I_2} \cdot S(x, 0, 2) + \frac{W_t}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2z}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1z}$$

$$z(x) := \frac{1}{E \cdot I} \cdot \left(\frac{R_{1z}}{6 \cdot I_2} \cdot S(x, 0, 3) + \frac{W_t}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2z}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1z} \cdot x + C_{2z}$$

Solving for constants:

$$z(0 \text{ mm}) = 0 \text{ mm} \quad z(L) = 0 \text{ mm}$$

$$C_{2z} := 0$$

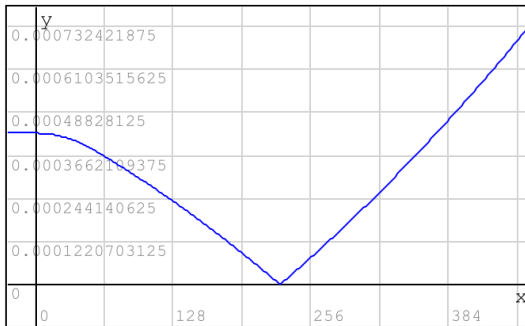
$$C_{1z} := \frac{\left(-\frac{1}{E} \right) \cdot \left(\frac{R_{1z}}{6 \cdot I_2} \cdot S(L, 0, 3) + \frac{W_t}{6 \cdot I_1} \cdot S(L, L_{gear}, 3) + \frac{R_{2z}}{6 \cdot I_2} \cdot S(L, L, 3) \right) + C_{2z}}{L} = 0.0004$$

$$\theta_y(x) := \frac{1}{E} \cdot \left(\frac{R_{1z}}{2 \cdot I_2} \cdot S(x, 0, 2) + \frac{W_t}{2 \cdot I_1} \cdot S(x, L_{gear}, 2) + \frac{R_{2z}}{2 \cdot I_2} \cdot S(x, L, 2) \right) + C_{1z}$$

$$z(x) := \frac{1}{E} \cdot \left(\frac{R_{1z}}{6 \cdot I_2} \cdot S(x, 0, 3) + \frac{W_t}{6 \cdot I_1} \cdot S(x, L_{gear}, 3) + \frac{R_{2z}}{6 \cdot I_2} \cdot S(x, L, 3) \right) + C_{1z} \cdot x + C_{2z}$$

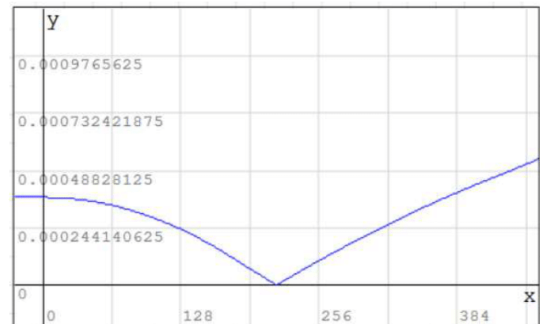
Total Angular Deflection

$$\theta(x) := \sqrt{(\theta_y(x))^2 + (\theta_z(x))^2}$$



$\theta(x \text{ mm}) \text{ rad}$

Figure 27: Total angular deflection for gear 1



$\theta(x \text{ mm}) \text{ rad}$

Figure 28: Total angular deflection for gear 2

KAG Output Shaft Analysis

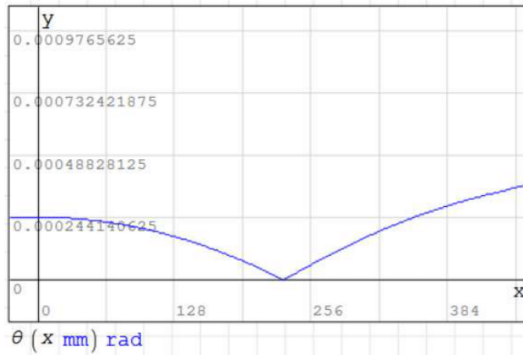


Figure 29: Total angular deflection for gear 3

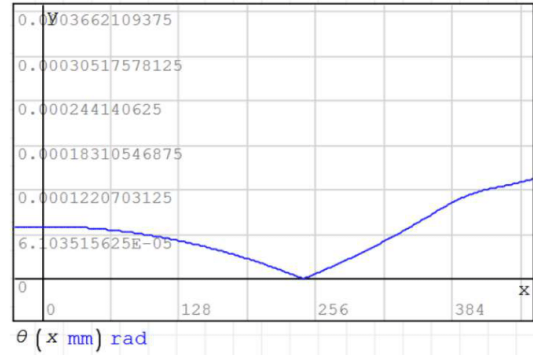


Figure 30: Total angular deflection for gear 4

Total Deflection

$$\delta(x) := \sqrt{z(x)^2 + y(x)^2}$$

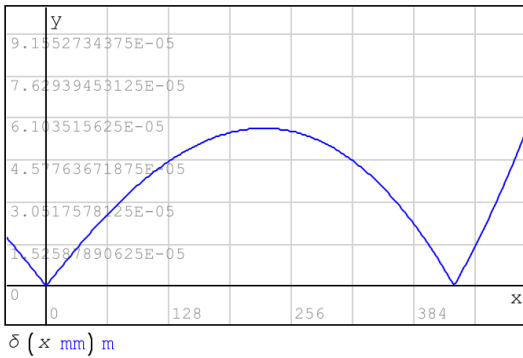


Figure 31: Total deflection for gear 1

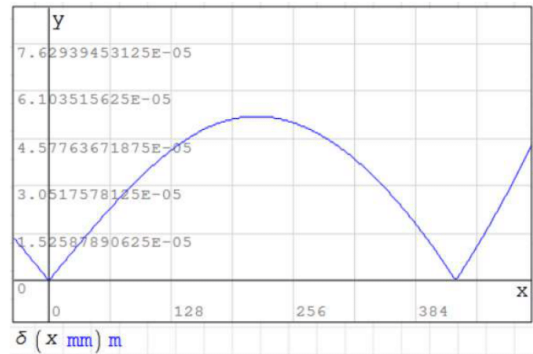


Figure 32: Total deflection for gear 2

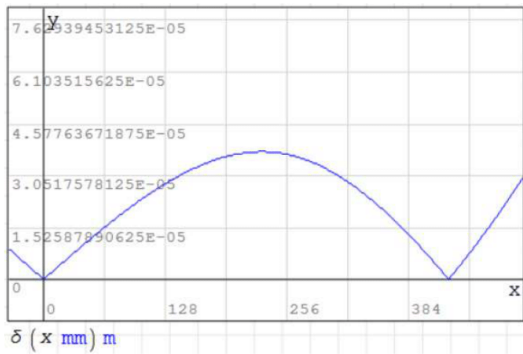


Figure 33: Total deflection for gear 3

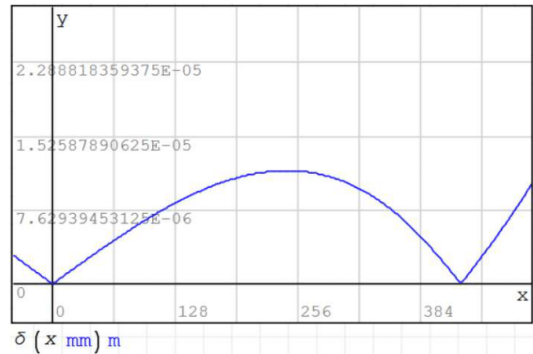


Figure 34: Total deflection for gear 4

Compare to evaluation criteria:

Deflection at the gear:

$$g_deflection := \delta(L_{gear}) = 0.0222 \text{ mm}$$

KAG Output Shaft Analysis

```
deflec_verify := if (g_deflection > 127 mm)
    "There is too much deflection at the gear "
else
    "Deflection at the gears was determined to be acceptable"
```

Angular Deflection at the gear:

$$g_angdeflec := \theta (L_{gear}) = 0.0004 \text{ rad}$$

```
g_angdeflec_verify := if (g_angdeflec > 0.0005 rad)
    "There is too much angular deflection at the gear"
else
    "Angular deflection at the gears was determined to be acceptable"
```

Angular Deflection at the bearings:

$$bear1_angdeflec := \theta (0 \text{ mm}) = 0.0004 \text{ rad}$$

$$bear2_angdeflec := \theta (L) = 0.0006 \text{ rad}$$

```
bear_angdeflec_verify := if (bear1_angdeflec > 0.004 rad)
    "There was determined to be too much angular deflection at bearing 1"
else
    if (bear2_angdeflec > 0.004 rad)
        "There was determined to be too much angular deflection at bearing 2"
    else
        "Angular deflection at the bearings was determined to be acceptable"
```

Angle of Twist:

$$\phi := \frac{T_{in}}{J_1 \cdot G} = 0.1737861 \frac{\text{deg}}{\text{m}}$$

```
angletwist_verify := if (phi > 3 \frac{\text{deg}}{\text{m}})
    "The angle of twist was determined to be too large"
else
    "The angle of twist was determined to be acceptable"
```

deflec_verify = "Deflection at the gears was determined to be acceptable"

bear_angdeflec_verify = "Angular deflection at the bearings was determined to be acceptable"

g_angdeflec_verify = "Angular deflection at the gears was determined to be acceptable"

angletwist_verify = "The angle of twist was determined to be acceptable"

d_check = "Diameter of 42 mm is large enough"

Alternating Bending Stress:

$$\sigma_a := \frac{K_f \cdot M_a \cdot \frac{d_1}{2}}{I_1} = 11.4165 \text{ MPa}$$

Mean Bending Stress:

$$\sigma_m := \frac{K_f \cdot M_m \cdot \frac{d_1}{2}}{I_1} = 0 \text{ MPa}$$

Alternating Torsional Stress:

$$\tau_a := \frac{K_{fs} \cdot T_a \cdot \frac{d_1}{2}}{J_1} = 6.6778 \text{ MPa}$$

Mean Torsional Stress:

$$\tau_m := \frac{K_{fs} \cdot T_m \cdot \frac{d_1}{2}}{J_1} = 6.6778 \text{ MPa}$$

2D Alternating Stress State (Von Mises):

2D Mean Stress State (Von Mises):

KAG Output Shaft Analysis

$$\sigma'_a := \sqrt{\sigma_a^2 + 3 \cdot \tau_a^2} = 16.2516 \text{ MPa}$$

$$\sigma'_m := \sqrt{\sigma_m^2 + 3 \cdot \tau_m^2} = 11.5662 \text{ MPa}$$

Calculated Safety Factors:

$$n_f := \frac{1}{\sqrt{\left(\frac{\sigma'_a}{S_e}\right)^2 + \left(\frac{\sigma'_m}{S_y}\right)^2}} = 16.9429$$

Fatigue Safety Factor

$$n_y := \frac{1}{\frac{\sigma'_a}{S_y} + \frac{\sigma'_m}{S_y}} = 21.0297$$

Yield Safety Factor

☐—Summarizing Minimum Diameters and verifying if Evaluation Criteria was met

Summary of values found

Gear	Minimum Diameter (mm)	Deflection at gear (mm)	Ang. Deflec. at gear (rad)	Max Ang. Deflec. at bearing (rad)	Angle of Twist (deg/m)
1	16.742	0.0222	0.0004	0.0006	0.1737861
2	18.4034	0.0502	0.0001	0.0005	0.1737861
3	17.5394	0.0370	$4.7444 \cdot 10^{-5}$	0.0003	0.1737861
4	15.6347	0.0061	$9.4047 \cdot 10^{-5}$	0.0001	0.1737861

Safety Factors

Gear	Fatigue Safety Factor	Yield Safety Factor
1	16.9429	21.0297
2	12.756	17.4565
3	14.7354	19.2158
4	20.8035	23.9123

Summary of whether or not evaluation criteria was met

Gear	Minimum Diameter	Deflection at gear	Ang. Deflec. at gear	Max Ang. Deflec. at bearing	Angle of Twist
1	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable
2	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable
3	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable
4	Acceptable	Acceptable	Acceptable	Acceptable	Acceptable

Keyway Analysis

Keyway Analysis

Inputs From Engine

Maximum Power

$$P_{eng} := 24.4 \text{ kW}$$

$$\omega_{eng} := 6500 \text{ rpm} = 680.6784 \text{ Hz}$$

Primary Reduction

$$r_0 := 2.272$$

Gear 1 Reduction

$$r_1 := 2$$

Torque Calculation

Torque before reduction

$$T_0 := \frac{P_{eng}}{\omega_{eng}} = 35.8466 \text{ N m}$$

Torque on input shaft

$$T_{in} := T_0 \cdot r_0 = 81.4435 \text{ N m}$$

Max torque on output shaft

$$T_{out} := T_{in} \cdot r_1 = 162.8869 \text{ N m}$$

Stainless Steel

$$t_y := 74.5 \text{ MPa}$$

Shaft Diameter

$$d_{shaft} := 0.042 \text{ m}$$

Key Dimensions

$$l := 76.2 \text{ mm}$$

$$w := 12 \text{ mm}$$

$$A := l \cdot w = 914.4000 \text{ mm}^2$$

Force on input shaft

$$F_{in} := \frac{T_{in}}{d_{shaft}} = 1939.130 \text{ N}$$

Force on output shaft

$$F_{out} := \frac{T_{out}}{d_{shaft}} = 3878.2597 \text{ N}$$

Stress experienced by key on input shaft

$$t_{in} := \frac{F_{in}}{A} = 2.120658 \text{ MPa}$$

Stress experienced by key on output shaft

$$t_{out} := \frac{F_{out}}{A} = 4.2413 \text{ MPa}$$

```
key_check := if t_in > t_y
              "Key on the input shaft will fail"
            else
              if t_out > t_y
                "Key on the output shaft will fail"
              else
                "Keys will NOT fail"
```

```
key_check = "Keys will NOT fail"
```

APPENDIX F: GEAR ANALYSIS

Final straight speed calcs

$$q := 0.85 \quad x := 1$$

$$m := 200 + 40 + 18 = 258$$

$$W_d := 183 \cdot \left(1.225 \cdot 0.5 \cdot 0.6 \cdot (26)^2 \right) = 45462.69$$

$$v_i := \sqrt{\frac{2 \cdot (30332 \cdot q \cdot 0.9)}{m}} = 13.4118$$

$$b := 7.44296969988$$

$$\int_{0.9}^b \sqrt{\frac{2 \cdot (30223 \cdot q \cdot x)}{m}} dx = 183.0018$$

$$E_{tot} := (30223 \cdot q \cdot b) - W_d = 145743.8523$$

$$v_f := \sqrt{\frac{2 \cdot E_{tot}}{m}} = 33.6125$$

First and Final Gear ratio calcs

Known values

130 80 17 tires --> radius of 0.32m

$$r := 0.32 \text{ m}$$

$$v_{max} := 33.6 \frac{\text{m}}{\text{s}}$$

$$v_{min} := 8.95 \frac{\text{m}}{\text{s}}$$

$$\omega_{maxpower} := 6000 \text{ rpm}$$

$$G_{initial} := 2.272$$

$$\omega_{engine} := \frac{\omega_{maxpower}}{G_{initial}} = 276.5486 \frac{\text{rad}}{\text{s}}$$

Calculate final gear ratio

$$\omega_{wheelmax} := \frac{v_{max}}{r} = 105 \frac{\text{rad}}{\text{s}}$$

$$G_{finalratio} := \frac{\omega_{engine}}{\omega_{wheelmax}} = 2.6338$$

Calculate initial gear ratio

$$\omega_{wheelmin} := \frac{v_{min}}{r} = 27.9688 \frac{\text{rad}}{\text{s}}$$

$$G_{initialratio} := \frac{\omega_{engine}}{\omega_{wheelmin}} = 9.8878$$

$$G_1 := \frac{1}{9.8878} = 0.1011$$

$$G_4 := \frac{1}{2.6388} = 0.379$$

Determine a common multiplier for each gear to maximize amount of time spent at high rpms

Use the geometric mean as final ratio before wheels to minimize large gear ratios

$$a := 3\sqrt{\frac{G_4}{G_1}} = 1.5532$$

$$b := \sqrt{G_1 \cdot G_4} = 0.1958$$

$$G_1 = 0.1011$$

$$G_2 := G_1 \cdot a = 0.1571$$

$$G_3 := G_2 \cdot a = 0.244$$

$$G_4 \cdot G_3 \cdot a = 0.379$$

determine the ideal adjusted gear ratios

approximate adjusted gear ratios and mean

$$\frac{G_1}{b} = 0.5166$$

$$G_{1a} := \frac{1}{2}$$

$$\frac{G_2}{b} = 0.8024$$

$$G_{2a} := \frac{4}{5}$$

$$c := \frac{1}{5}$$

$$\frac{G_3}{b} = 1.2463$$

$$G_{3a} := \frac{5}{4}$$

$$\frac{G_4}{b} = 1.9357$$

$$G_{4a} := 2$$

check error in calculated top speed for each gear

$$\text{final speed function: } v(G) := \frac{6000 \text{ rpm}}{2.272} \cdot 0.32 \text{ m} \cdot G$$

$$\varepsilon_{g1} := \frac{v(G_{1a} \cdot c) - v(G_1)}{v(G_1)} = -0.0112$$

$$\varepsilon_{g2} := \frac{v(G_{2a} \cdot c) - v(G_2)}{v(G_2)} = 0.0186$$

$$\varepsilon_{g3} := \frac{v(G_{3a} \cdot c) - v(G_3)}{v(G_3)} = 0.0247$$

$$\varepsilon_{g4} := \frac{v(G_{4a} \cdot c) - v(G_4)}{v(G_4)} = 0.0555$$

the maximum error in top speed of any gear was determined to be 5.6% which was determined to be within the acceptable range

1. Engine Inputs

$$P := 24.4 \text{ kW}$$

$$\omega := 6500 \text{ rpm} = 680.6784 \frac{\text{rad}}{\text{s}}$$

2. Gear Parameters**Pressure Angle:**

$$\phi := 20 \text{ deg}$$

Face Width

$$F_{i1} := 76.2 \text{ mm}$$

$$F_{i2} := 76.2 \text{ mm}$$

$$F_{i3} := 76.2 \text{ mm}$$

$$F_{i4} := 76.2 \text{ mm}$$

$$F_{o1} := 76.2 \text{ mm}$$

$$F_{o2} := 76.2 \text{ mm}$$

$$F_{o3} := 76.2 \text{ mm}$$

$$F_{o4} := 76.2 \text{ mm}$$

Quality of Gears:

$$Q_v := 9$$

Number of Gear Teeth

$$N_{i1} := 42$$

$$N_{i2} := 35$$

$$N_{i3} := 28$$

$$N_{i4} := 21$$

$$N_{o1} := 21$$

$$N_{o2} := 28$$

$$N_{o3} := 35$$

$$N_{o4} := 42$$

Selected Gear Ratios

$$r_p := 2.272$$

$$r_1 := \frac{N_{o1}}{N_{i1}} = 0.5$$

$$r_2 := \frac{N_{o2}}{N_{i2}} = 0.8$$

$$r_3 := \frac{N_{o3}}{N_{i3}} = 1.25$$

$$r_4 := \frac{N_{o4}}{N_{i4}} = 2$$

Gear Pitch Circle Diameters

$$d_{i1} := 355.6 \text{ mm}$$

$$d_{i2} := 296.342 \text{ mm}$$

$$d_{i3} := 237.058 \text{ mm}$$

$$d_{i4} := 177.8 \text{ mm}$$

$$d_{o1} := 177.8 \text{ mm}$$

$$d_{o2} := 237.058 \text{ mm}$$

$$d_{o3} := 296.342 \text{ mm}$$

$$d_{o4} := 355.6 \text{ mm}$$

Module

$$m_1 := \frac{d_{i1}}{N_{i1}} = 8.4667 \text{ mm}$$

$$m_2 := \frac{d_{i2}}{N_{i2}} = 8.4669 \text{ mm}$$

$$m_3 := \frac{d_{i3}}{N_{i3}} = 8.4664 \text{ mm}$$

$$m_4 := \frac{d_{i4}}{N_{i4}} = 8.4667 \text{ mm}$$

3. AGMA Correction Factors and Calculations**Torque**

$$T_{in} := \frac{P}{\omega} \cdot r_p = 81.4435 \text{ N m}$$

Tangential Forces

$$W_{t1} := 2 \cdot \frac{T_{in}}{d_{i1}} = 458.0622 \text{ N}$$

$$W_{t2} := 2 \cdot \frac{T_{in}}{d_{i2}} = 549.6585 \text{ N}$$

$$W_{t3} := 2 \cdot \frac{T_{in}}{d_{i3}} = 687.1184 \text{ N}$$

$$W_{t4} := 2 \cdot \frac{T_{in}}{d_{i4}} = 916.1243 \text{ N}$$

Radial Forces

$$W_{r1} := W_{t1} \cdot \tan(\phi) = 166.721 \text{ N}$$

$$W_{r2} := W_{t2} \cdot \tan(\phi) = 200.0593 \text{ N}$$

$$W_{r3} := W_{t3} \cdot \tan(\phi) = 250.0906 \text{ N}$$

$$W_{r4} := W_{t4} \cdot \tan(\phi) = 333.442 \text{ N}$$

AGMA Assumptions

- No Interference
- Teeth are not pointed
- There is a non-zero backlash
- Fillets are standard, assumed smooth, and produced during the generation process
- Friction Forces are negligible

KAG Assumptions

- 4 stroke single cylinder engine under moderate shock
- Solid disk gears
- 90% reliability
- Operating temperature of 195 F
- Transmission designed for 10^7 cycles
- Gears made from SCM415 Steel - surface hardness 55 – 64 HRC

Application Factor

$$\begin{array}{llll}
 K_{a_i1} := 1.75 & K_{a_o1} := 1.75 & C_{a_i1} := K_{a_i1} = 1.75 & C_{a_o1} := K_{a_o1} = 1.75 \\
 K_{a_i2} := 1.75 & K_{a_o2} := 1.75 & C_{a_i2} := K_{a_i2} = 1.75 & C_{a_o2} := K_{a_o2} = 1.75 \\
 K_{a_i3} := 1.75 & K_{a_o3} := 1.75 & C_{a_i3} := K_{a_i3} = 1.75 & C_{a_o3} := K_{a_o3} = 1.75 \\
 K_{a_i4} := 1.75 & K_{a_o4} := 1.75 & C_{a_i4} := K_{a_i4} = 1.75 & C_{a_o4} := K_{a_o4} = 1.75
 \end{array}$$

Load Distribution Factor

$$\begin{array}{llll}
 K_{m_i1} := 1.6 & K_{m_o1} := 1.6 & C_{m_i1} := K_{m_i1} = 1.6 & C_{m_o1} := K_{m_o1} = 1.6 \\
 K_{m_i2} := 1.6 & K_{m_o2} := 1.6 & C_{m_i2} := K_{m_i2} = 1.6 & C_{m_o2} := K_{m_o2} = 1.6 \\
 K_{m_i3} := 1.6 & K_{m_o3} := 1.6 & C_{m_i3} := K_{m_i3} = 1.6 & C_{m_o3} := K_{m_o3} = 1.6 \\
 K_{m_i4} := 1.6 & K_{m_o4} := 1.6 & C_{m_i4} := K_{m_i4} = 1.6 & C_{m_o4} := K_{m_o4} = 1.6
 \end{array}$$

Dynamic Factor

$$\begin{array}{llll}
 B := \frac{(12 - Q_v)^{\frac{2}{3}}}{4} = 0.52 & v_{t1} := \frac{d_{i1}}{2} \cdot \frac{\omega}{r_p} = 53.2679 \frac{\text{m}}{\text{s}} & v_{t3} := \frac{d_{i3}}{2} \cdot \frac{\omega}{r_p} = 35.5106 \frac{\text{m}}{\text{s}} \\
 A := 50 + 56 \cdot (1 - B) = 76.8788 & v_{t2} := \frac{d_{i2}}{2} \cdot \frac{\omega}{r_p} = 44.3912 \frac{\text{m}}{\text{s}} & v_{t4} := \frac{d_{i4}}{2} \cdot \frac{\omega}{r_p} = 26.6339 \frac{\text{m}}{\text{s}}
 \end{array}$$

$$\begin{array}{ll}
 K_{v1} := \frac{50}{50 + \sqrt{200 \frac{\text{S}}{\text{m}} \cdot v_{t1}}} = 0.3263 & C_{v1} := K_{v1} = 0.3263 \\
 K_{v2} := \frac{50}{50 + \sqrt{200 \frac{\text{S}}{\text{m}} \cdot v_{t2}}} = 0.3467 & C_{v2} := K_{v2} = 0.3467 \\
 K_{v3} := \frac{50}{50 + \sqrt{200 \frac{\text{S}}{\text{m}} \cdot v_{t3}}} = 0.3724 & C_{v3} := K_{v3} = 0.3724 \\
 K_{v4} := \frac{50}{50 + \sqrt{200 \frac{\text{S}}{\text{m}} \cdot v_{t4}}} = 0.4066 & C_{v4} := K_{v4} = 0.4066
 \end{array}$$

Size Factor

$K_{s_{i1}} := 1$	$K_{s_{o1}} := 1$	$C_{s_{i1}} := K_{s_{i1}} = 1$	$C_{s_{o1}} := K_{s_{o1}} = 1$
$K_{s_{i2}} := 1$	$K_{s_{o2}} := 1$	$C_{s_{i2}} := K_{s_{i2}} = 1$	$C_{s_{o2}} := K_{s_{o2}} = 1$
$K_{s_{i3}} := 1$	$K_{s_{o3}} := 1$	$C_{s_{i3}} := K_{s_{i3}} = 1$	$C_{s_{o3}} := K_{s_{o3}} = 1$
$K_{s_{i4}} := 1$	$K_{s_{o4}} := 1$	$C_{s_{i4}} := K_{s_{i4}} = 1$	$C_{s_{o4}} := K_{s_{o4}} = 1$

Geometry Factor, J

$J_{i1} := 0.26$	$J_{o1} := 0.24$
$J_{i2} := 0.26$	$J_{o2} := 0.25$
$J_{i3} := 0.25$	$J_{o3} := 0.26$
$J_{i4} := 0.24$	$J_{o4} := 0.26$

Rim Thickness Factor

$K_{B_{i1}} := 1$	$K_{B_{o1}} := 1$
$K_{B_{i2}} := 1$	$K_{B_{o2}} := 1$
$K_{B_{i3}} := 1$	$K_{B_{o3}} := 1$
$K_{B_{i4}} := 1$	$K_{B_{o4}} := 1$

Idler Factor

$K_{I_{i1}} := 1$	$K_{I_{o1}} := 1$
$K_{I_{i2}} := 1$	$K_{I_{o2}} := 1$
$K_{I_{i3}} := 1$	$K_{I_{o3}} := 1$
$K_{I_{i4}} := 1$	$K_{I_{o4}} := 1$

Geometry Factor, I

$I_{i1} := \left(\frac{\sin(\phi) \cdot \cos(\phi)}{2} \right) \cdot \left(\frac{N_{o1}}{N_{o1} + N_{i1}} \right) = 0.0536$	$I_{o1} := I_{i1} = 0.0536$
$I_{i2} := \left(\frac{\sin(\phi) \cdot \cos(\phi)}{2} \right) \cdot \left(\frac{N_{o2}}{N_{o2} + N_{i2}} \right) = 0.0714$	$I_{o2} := I_{i2} = 0.0714$
$I_{i3} := \left(\frac{\sin(\phi) \cdot \cos(\phi)}{2} \right) \cdot \left(\frac{N_{o3}}{N_{o3} + N_{i3}} \right) = 0.0893$	$I_{o3} := I_{i3} = 0.0893$
$I_{i4} := \left(\frac{\sin(\phi) \cdot \cos(\phi)}{2} \right) \cdot \left(\frac{N_{o4}}{N_{o4} + N_{i4}} \right) = 0.1071$	$I_{o4} := I_{i4} = 0.1071$

Elastic Coefficient(Steel)

$C_{p_{i1}} := 191 \text{ MPa}^{0.5}$	$C_{p_{o1}} := 191 \text{ MPa}^{0.5}$
$C_{p_{i2}} := 191 \text{ MPa}^{0.5}$	$C_{p_{o2}} := 191 \text{ MPa}^{0.5}$
$C_{p_{i3}} := 191 \text{ MPa}^{0.5}$	$C_{p_{o3}} := 191 \text{ MPa}^{0.5}$
$C_{p_{i4}} := 191 \text{ MPa}^{0.5}$	$C_{p_{o4}} := 191 \text{ MPa}^{0.5}$

Surface Finish Factor

$C_{f_{i1}} := 1$	$C_{f_{o1}} := 1$
$C_{f_{i2}} := 1$	$C_{f_{o2}} := 1$
$C_{f_{i3}} := 1$	$C_{f_{o3}} := 1$
$C_{f_{i4}} := 1$	$C_{f_{o4}} := 1$

AGMA Uncorrected Bending Strength & Wear Strength [SCM415 - Surface Hardness(55-64 HRC)]

$S'_{fb_{i1}} := 520 \text{ MPa}$	$S'_{fb_{o1}} := 520 \text{ MPa}$	$S'_{fc_{i1}} := 1300 \text{ MPa}$	$S'_{fc_{o1}} := 1300 \text{ MPa}$
$S'_{fb_{i2}} := 520 \text{ MPa}$	$S'_{fb_{o2}} := 520 \text{ MPa}$	$S'_{fc_{i2}} := 1300 \text{ MPa}$	$S'_{fc_{o2}} := 1300 \text{ MPa}$
$S'_{fb_{i3}} := 520 \text{ MPa}$	$S'_{fb_{o3}} := 520 \text{ MPa}$	$S'_{fc_{i3}} := 1300 \text{ MPa}$	$S'_{fc_{o3}} := 1300 \text{ MPa}$
$S'_{fb_{i4}} := 520 \text{ MPa}$	$S'_{fb_{o4}} := 520 \text{ MPa}$	$S'_{fc_{i4}} := 1300 \text{ MPa}$	$S'_{fc_{o4}} := 1300 \text{ MPa}$

Temperature Factor

$T_f := 195$

$K_T := \frac{460 + T_f}{620} = 1.0565$

$C_T := K_T = 1.0565$

Reliability Factor

$K_R := 0.85$

$C_R := K_R = 0.85$

Hardness Factor

$C_H := 1$

Input Shaft Rotation Calculations

$$O_{motor} := \omega \cdot 0.8 = 5200 \text{ rpm}$$

$$\omega_i := \frac{O_{motor}}{r_p} = 2288.7324 \text{ rpm}$$

$$N_r := 2$$

$$T_r := 10 \text{ min}$$

Shaft Cycles

$$\xi_{G1} := \omega_i \cdot \frac{N_r}{4} \cdot T_r = 71902.6488$$

$$\xi_{G2} := \omega_i \cdot \frac{N_r}{4} \cdot T_r = 71902.6488$$

$$\xi_{G3} := \omega_i \cdot \frac{N_r}{4} \cdot T_r \cdot \frac{N_{i3}}{N_{o3}} = 5.7522 \cdot 10^4$$

$$\xi_{G4} := \omega_i \cdot \frac{N_r}{4} \cdot T_r \cdot \frac{N_{i4}}{N_{o4}} = 3.5951 \cdot 10^4$$

Life Factor $\xi < 10^7$ & HB = 550

$$C_{L1} := 2.466 \cdot \xi_{G1}^{-0.056} = 1.3183 \quad K_{L1} := 9.4518 \cdot \xi_{G1}^{-0.148} = 1.806$$

$$C_{L2} := 2.466 \cdot \xi_{G2}^{-0.056} = 1.3183 \quad K_{L2} := 9.4518 \cdot \xi_{G2}^{-0.148} = 1.806$$

$$C_{L3} := 2.466 \cdot \xi_{G3}^{-0.056} = 1.3349 \quad K_{L3} := 9.4518 \cdot \xi_{G3}^{-0.148} = 1.8666$$

$$C_{L4} := 2.466 \cdot \xi_{G4}^{-0.056} = 1.3705 \quad K_{L4} := 9.4518 \cdot \xi_{G4}^{-0.148} = 2.0011$$

Bending Stresses

$$\sigma_{b_{i1}} := \frac{W_{t1}}{F_{i1} \cdot m_1 \cdot J_{i1}} \cdot K_{a_{i1}} \cdot \frac{K_{m_{i1}}}{K_{v1}} \cdot K_{s_{i1}} \cdot K_{B_{i1}} \cdot K_{I_{i1}} = 23.4302 \text{ MPa}$$

$$\sigma_{b_{i2}} := \frac{W_{t2}}{F_{i2} \cdot m_2 \cdot J_{i2}} \cdot K_{a_{i2}} \cdot \frac{K_{m_{i2}}}{K_{v2}} \cdot K_{s_{i2}} \cdot K_{B_{i2}} \cdot K_{I_{i2}} = 26.4647 \text{ MPa}$$

$$\sigma_{b_{i3}} := \frac{W_{t3}}{F_{i3} \cdot m_3 \cdot J_{i3}} \cdot K_{a_{i3}} \cdot \frac{K_{m_{i3}}}{K_{v3}} \cdot K_{s_{i3}} \cdot K_{B_{i3}} \cdot K_{I_{i3}} = 32.0347 \text{ MPa}$$

$$\sigma_{b_{i4}} := \frac{W_{t4}}{F_{i4} \cdot m_4 \cdot J_{i4}} \cdot K_{a_{i4}} \cdot \frac{K_{m_{i4}}}{K_{v4}} \cdot K_{s_{i4}} \cdot K_{B_{i4}} \cdot K_{I_{i4}} = 40.7489 \text{ MPa}$$

$$\sigma_{b_{o1}} := \frac{W_{t1}}{F_{o1} \cdot m_1 \cdot J_{o1}} \cdot K_{a_{o1}} \cdot \frac{K_{m_{o1}}}{K_{v1}} \cdot K_{s_{o1}} \cdot K_{B_{o1}} \cdot K_{I_{o1}} = 25.3827 \text{ MPa}$$

$$\sigma_{b_{o2}} := \frac{W_{t2}}{F_{o2} \cdot m_2 \cdot J_{o2}} \cdot K_{a_{o2}} \cdot \frac{K_{m_{o2}}}{K_{v2}} \cdot K_{s_{o2}} \cdot K_{B_{o2}} \cdot K_{I_{o2}} = 27.5232 \text{ MPa}$$

$$\sigma_{b_{o3}} := \frac{W_{t3}}{F_{o3} \cdot m_3 \cdot J_{o3}} \cdot K_{a_{o3}} \cdot \frac{K_{m_{o3}}}{K_{v3}} \cdot K_{s_{o3}} \cdot K_{B_{o3}} \cdot K_{I_{o3}} = 30.8026 \text{ MPa}$$

$$\sigma_{b_{o4}} := \frac{W_{t4}}{F_{o4} \cdot m_4 \cdot J_{o4}} \cdot K_{a_{o4}} \cdot \frac{K_{m_{o4}}}{K_{v4}} \cdot K_{s_{o4}} \cdot K_{B_{o4}} \cdot K_{I_{o4}} = 37.6143 \text{ MPa}$$

Contact Stresses

$$\sigma_{c_i1} := C_{p_i1} \cdot \sqrt{\frac{W_{t1}}{F_{i1} \cdot I_{i1} \cdot d_{i1}} \cdot C_{a_i1} \cdot \frac{C_{m_i1}}{C_{v1}} \cdot C_{s_i1} \cdot C_{f_i1}} = 314.2971 \text{ MPa}$$

$$\sigma_{c_i2} := C_{p_i2} \cdot \sqrt{\frac{W_{t2}}{F_{i2} \cdot I_{i2} \cdot d_{i2}} \cdot C_{a_i2} \cdot \frac{C_{m_i2}}{C_{v2}} \cdot C_{s_i2} \cdot C_{f_i2}} = 316.8886 \text{ MPa}$$

$$\sigma_{c_i3} := C_{p_i3} \cdot \sqrt{\frac{W_{t3}}{F_{i3} \cdot I_{i3} \cdot d_{i3}} \cdot C_{a_i3} \cdot \frac{C_{m_i3}}{C_{v3}} \cdot C_{s_i3} \cdot C_{f_i3}} = 341.8747 \text{ MPa}$$

$$\sigma_{c_i4} := C_{p_i4} \cdot \sqrt{\frac{W_{t4}}{F_{i4} \cdot I_{i4} \cdot d_{i4}} \cdot C_{a_i4} \cdot \frac{C_{m_i4}}{C_{v4}} \cdot C_{s_i4} \cdot C_{f_i4}} = 398.2254 \text{ MPa}$$

$$\sigma_{c_o1} := C_{p_o1} \cdot \sqrt{\frac{W_{t1}}{F_{o1} \cdot I_{o1} \cdot d_{o1}} \cdot C_{a_o1} \cdot \frac{C_{m_o1}}{C_{v1}} \cdot C_{s_o1} \cdot C_{f_o1}} = 444.4832 \text{ MPa}$$

$$\sigma_{c_o2} := C_{p_o2} \cdot \sqrt{\frac{W_{t2}}{F_{o2} \cdot I_{o2} \cdot d_{o2}} \cdot C_{a_o2} \cdot \frac{C_{m_o2}}{C_{v2}} \cdot C_{s_o2} \cdot C_{f_o2}} = 354.3039 \text{ MPa}$$

$$\sigma_{c_o3} := C_{p_o3} \cdot \sqrt{\frac{W_{t3}}{F_{o3} \cdot I_{o3} \cdot d_{o3}} \cdot C_{a_o3} \cdot \frac{C_{m_o3}}{C_{v3}} \cdot C_{s_o3} \cdot C_{f_o3}} = 305.772 \text{ MPa}$$

$$\sigma_{c_o4} := C_{p_o4} \cdot \sqrt{\frac{W_{t4}}{F_{o4} \cdot I_{o4} \cdot d_{o4}} \cdot C_{a_o4} \cdot \frac{C_{m_o4}}{C_{v4}} \cdot C_{s_o4} \cdot C_{f_o4}} = 281.5878 \text{ MPa}$$

Bending Strengths

$$S_{fb_i1} := \frac{K_{L1}}{K_T \cdot K_R} \cdot S'_{fb_i1} = 1045.8059 \text{ MPa}$$

$$S_{fb_o1} := \frac{K_{L1}}{K_T \cdot K_R} \cdot S'_{fb_o1} = 1045.8059 \text{ MPa}$$

$$S_{fb_i2} := \frac{K_{L2}}{K_T \cdot K_R} \cdot S'_{fb_i2} = 1045.8059 \text{ MPa}$$

$$S_{fb_o2} := \frac{K_{L2}}{K_T \cdot K_R} \cdot S'_{fb_o2} = 1045.8059 \text{ MPa}$$

$$S_{fb_i3} := \frac{K_{L3}}{K_T \cdot K_R} \cdot S'_{fb_i3} = 1080.9205 \text{ MPa}$$

$$S_{fb_o3} := \frac{K_{L3}}{K_T \cdot K_R} \cdot S'_{fb_o3} = 1080.9205 \text{ MPa}$$

$$S_{fb_i4} := \frac{K_{L4}}{K_T \cdot K_R} \cdot S'_{fb_i4} = 1158.7868 \text{ MPa}$$

$$S_{fb_o4} := \frac{K_{L4}}{K_T \cdot K_R} \cdot S'_{fb_o4} = 1158.7868 \text{ MPa}$$

Surface Strengths

$$S_{fc_i1} := \frac{C_{L1} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_i1} = 1908.4913 \text{ MPa}$$

$$S_{fc_o1} := \frac{C_{L1} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_o1} = 1908.4913 \text{ MPa}$$

$$S_{fc_i2} := \frac{C_{L2} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_i2} = 1908.4913 \text{ MPa}$$

$$S_{fc_o2} := \frac{C_{L2} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_o2} = 1908.4913 \text{ MPa}$$

$$S_{fc_i3} := \frac{C_{L3} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_i3} = 1932.4895 \text{ MPa}$$

$$S_{fc_o3} := \frac{C_{L3} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_o3} = 1932.4895 \text{ MPa}$$

$$S_{fc_i4} := \frac{C_{L4} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_i4} = 1984.0283 \text{ MPa}$$

$$S_{fc_o4} := \frac{C_{L4} \cdot C_H}{C_T \cdot C_R} \cdot S'_{fc_o4} = 1984.0283 \text{ MPa}$$

Bending Safety Factors

$$n_{b_i1} := \frac{S_{fb_i1}}{\sigma_{b_i1}} = 44.6349$$

$$n_{b_o1} := \frac{S_{fb_o1}}{\sigma_{b_o1}} = 41.2015$$

$$n_{b_i2} := \frac{S_{fb_i2}}{\sigma_{b_i2}} = 39.5171$$

$$n_{b_o2} := \frac{S_{fb_o2}}{\sigma_{b_o2}} = 37.9972$$

$$n_{b_i3} := \frac{S_{fb_i3}}{\sigma_{b_i3}} = 33.7422$$

$$n_{b_o3} := \frac{S_{fb_o3}}{\sigma_{b_o3}} = 35.0919$$

$$n_{b_i4} := \frac{S_{fb_i4}}{\sigma_{b_i4}} = 28.4373$$

$$n_{b_o4} := \frac{S_{fb_o4}}{\sigma_{b_o4}} = 30.8071$$

Wear Safety Factors

$$n_{o_i1} := \left(\frac{S_{fc_i1}}{\sigma_{c_i1}} \right)^2 = 36.8722$$

$$n_{o_o1} := \left(\frac{S_{fc_o1}}{\sigma_{c_o1}} \right)^2 = 18.4361$$

$$n_{o_i2} := \left(\frac{S_{fc_i2}}{\sigma_{c_i2}} \right)^2 = 36.2716$$

$$n_{o_o2} := \left(\frac{S_{fc_o2}}{\sigma_{c_o2}} \right)^2 = 29.0154$$

$$n_{o_i3} := \left(\frac{S_{fc_i3}}{\sigma_{c_i3}} \right)^2 = 31.9522$$

$$n_{o_o3} := \left(\frac{S_{fc_o3}}{\sigma_{c_o3}} \right)^2 = 39.9428$$

$$n_{o_i4} := \left(\frac{S_{fc_i4}}{\sigma_{c_i4}} \right)^2 = 24.8221$$

$$n_{o_o4} := \left(\frac{S_{fc_o4}}{\sigma_{c_o4}} \right)^2 = 49.6441$$

APPENDIX G: BEARING ANALYSIS

10 Dec 2021 15:56:59 - Bearing analysis.sm

KAG Bearing Analysis

Maximum force in y-direction
(Based on maximum reaction forces
in shaft analysis):

$$R_y := 278.0122 \text{ N}$$

Reliability factor:

$$K_r := 1 \text{ cycle}$$

Input Bearing:

Speed:

$$W_{output} := 13000 \frac{\text{cycle}}{\text{min}}$$

Fatigue Life:

$$L_{input} := S \cdot W_{input} = 3.9 \cdot 10^6 \text{ cycle}$$

Critical Force

$$Cr_{input} := P \cdot \left(\frac{L_{input}}{10^6 \cdot K_r} \right)^{\frac{1}{a}} = 1279.4804 \text{ N}$$

Maximum force in z-direction:
(Based on maximum reaction forces
in shaft analysis):

$$R_z := 763.8321 \text{ N}$$

Constant due to ball bearings:

$$a := 3$$

Output Bearing:

Speed:

$$W_{input} := 6500 \frac{\text{cycle}}{\text{min}}$$

Fatigue Life:

$$L_{output} := S \cdot W_{output} = 7.8 \cdot 10^6 \text{ cycle}$$

Critical force:

$$Cr_{output} := P \cdot \left(\frac{L_{output}}{10^6 \cdot K_r} \right)^{\frac{1}{a}} = 1612.0443 \text{ N}$$

Lifetime:

$$S := 10 \text{ hr}$$

Resultant radial force:

$$P := \sqrt{R_y^2 + R_z^2} = 812.8532 \text{ N}$$

APPENDIX H: DRAWING PACKAGE

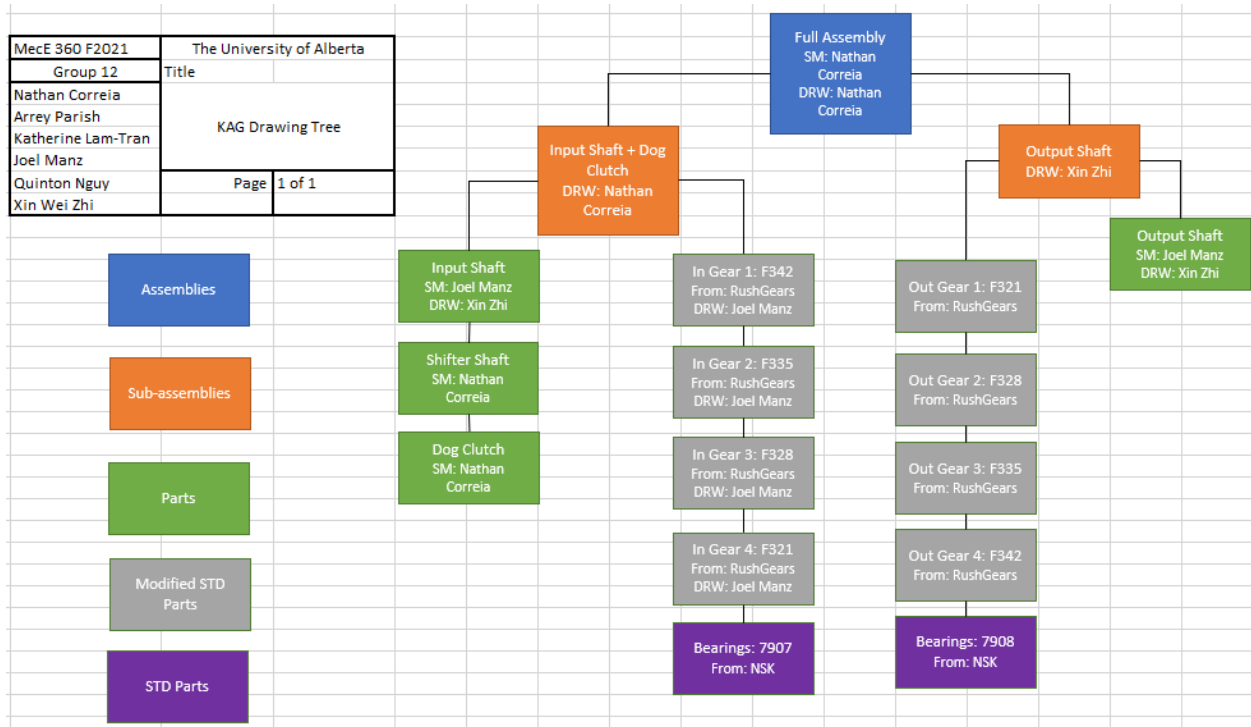
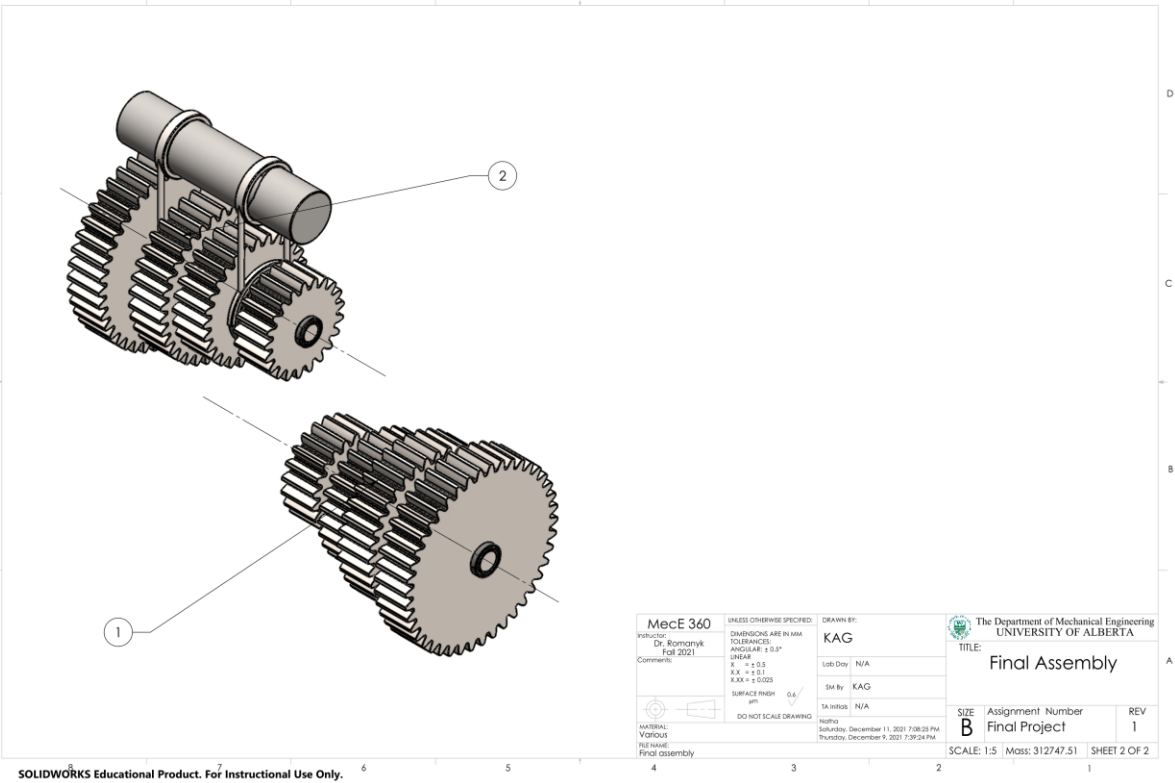
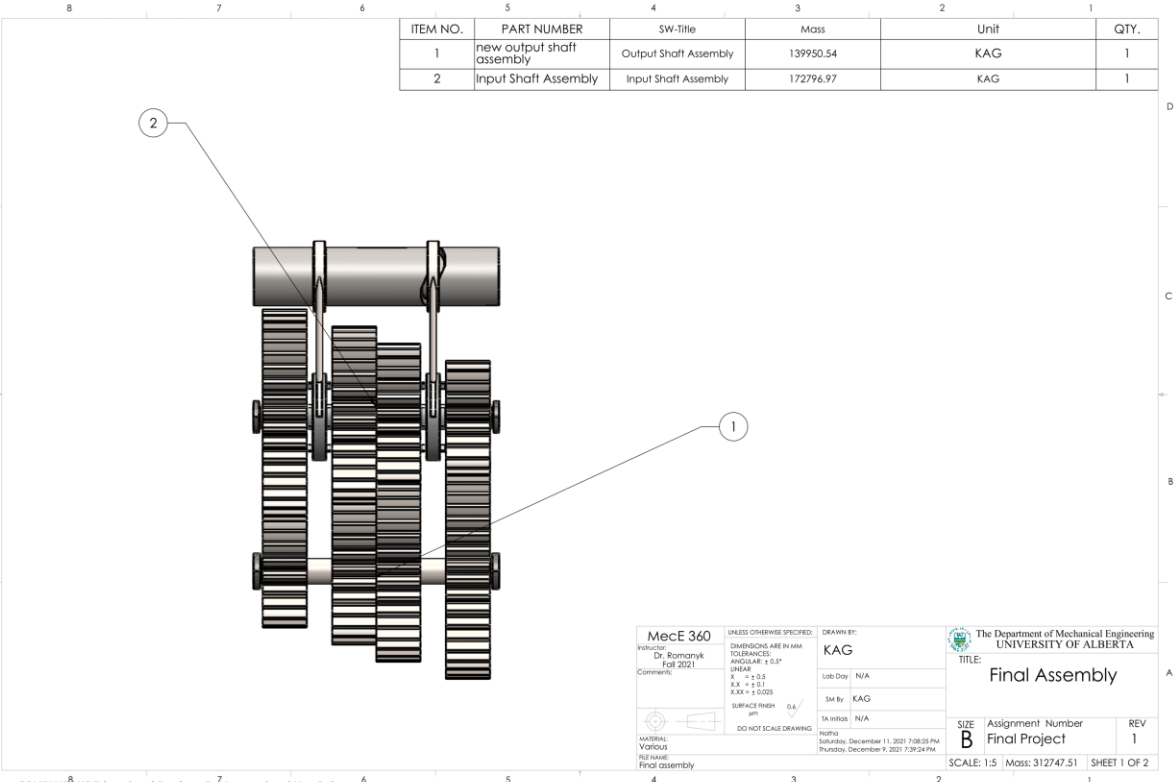
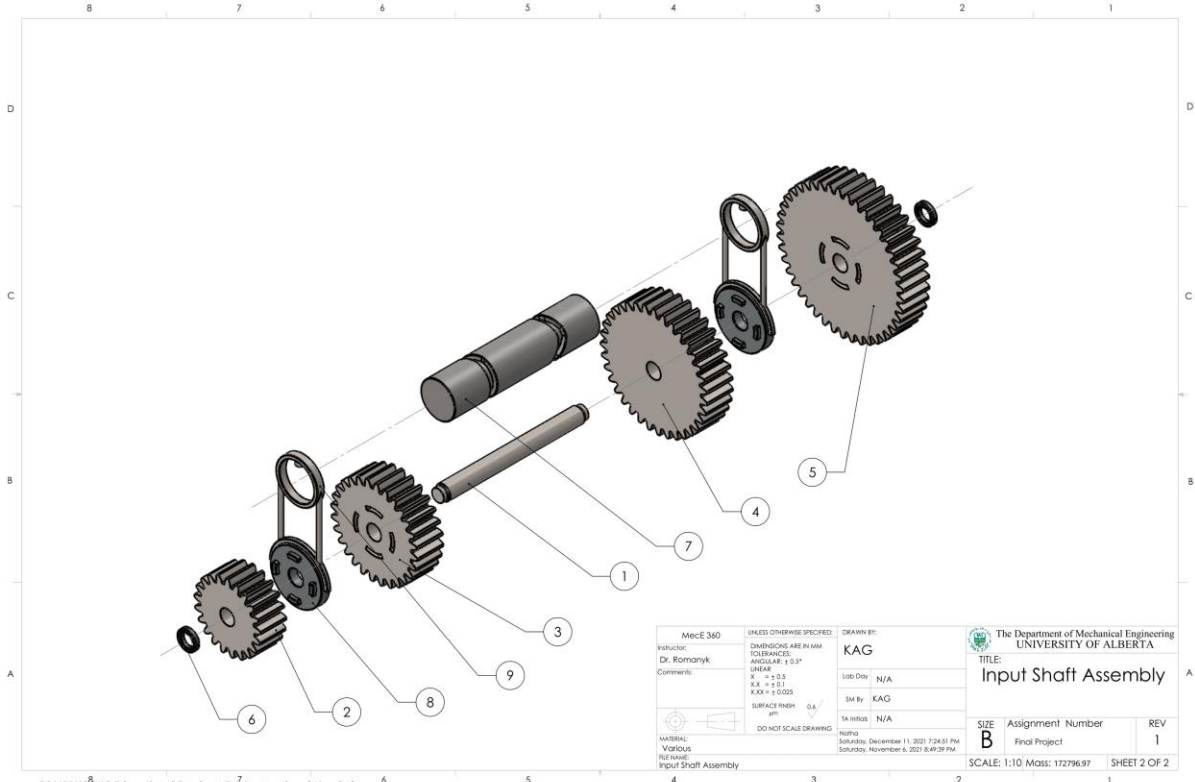


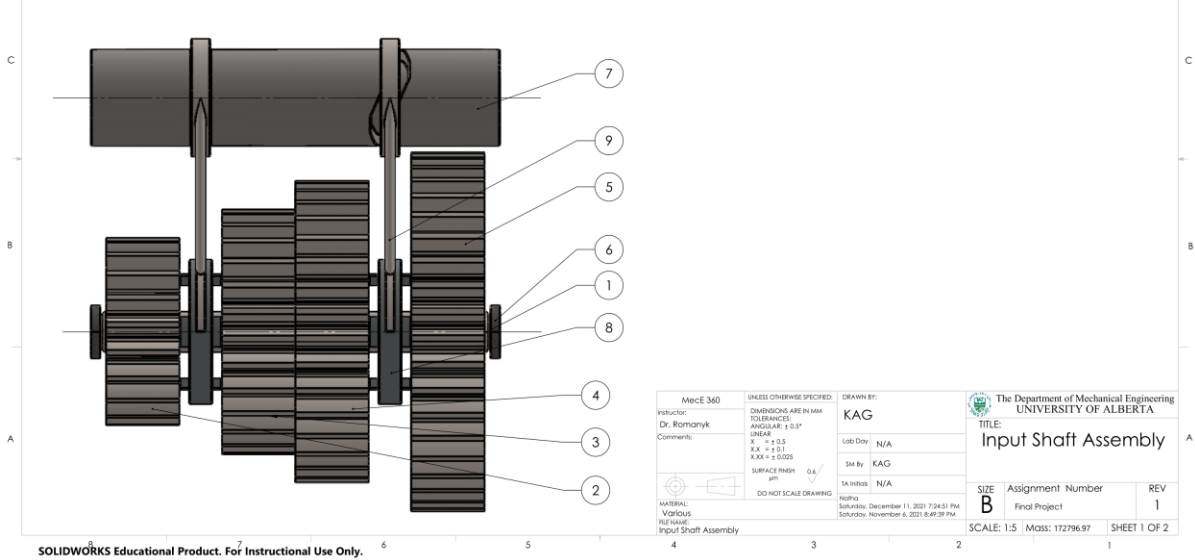
Figure 77: KAG Drawing Tree



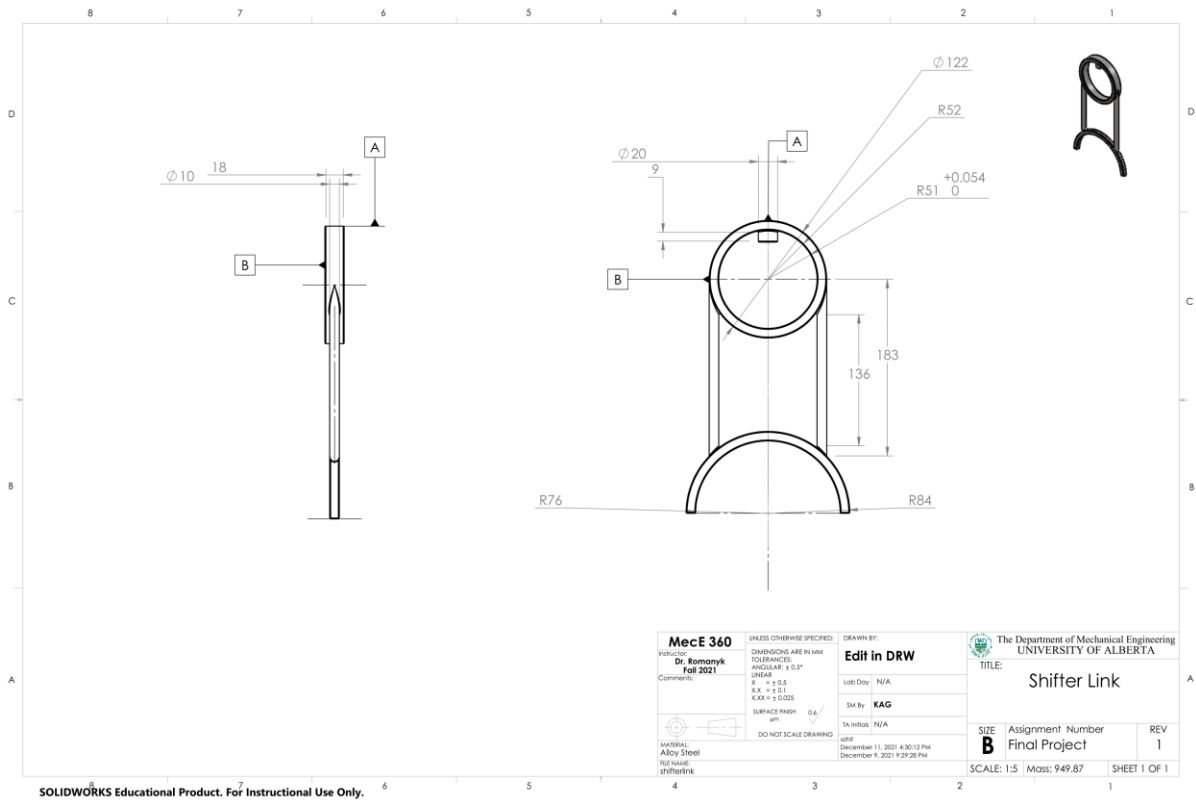
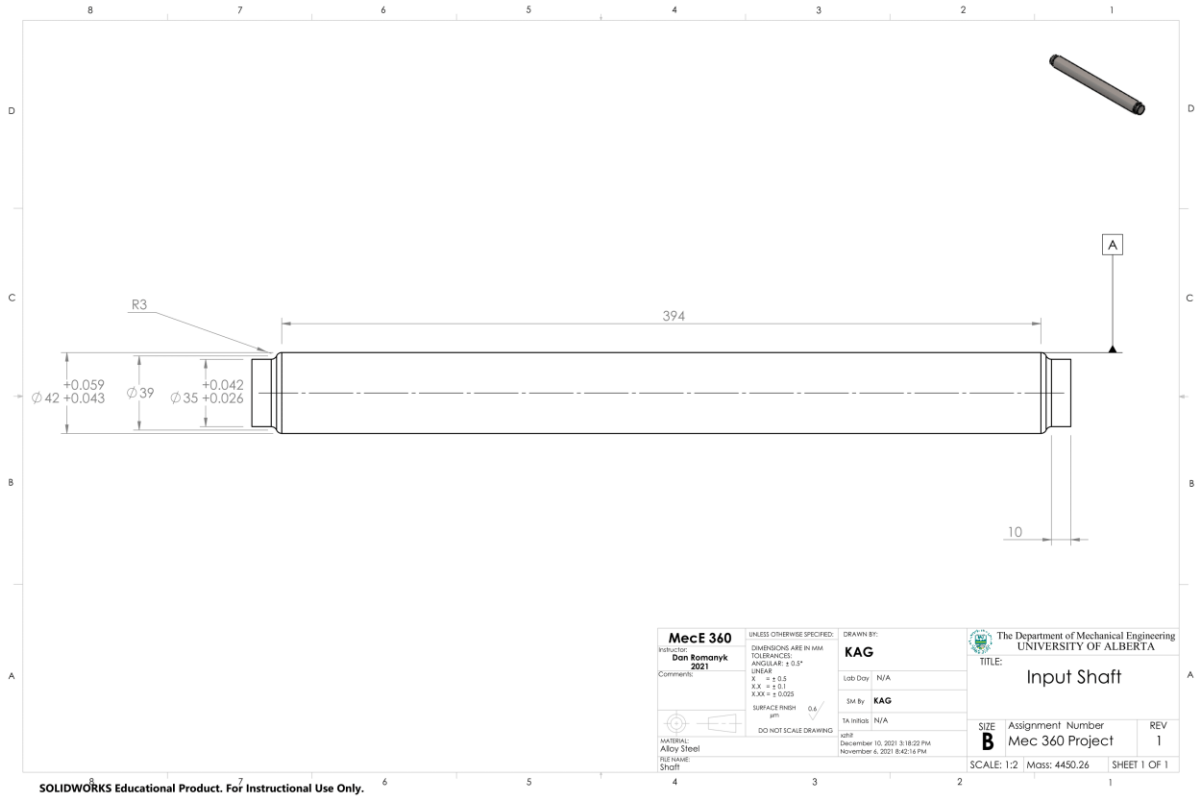


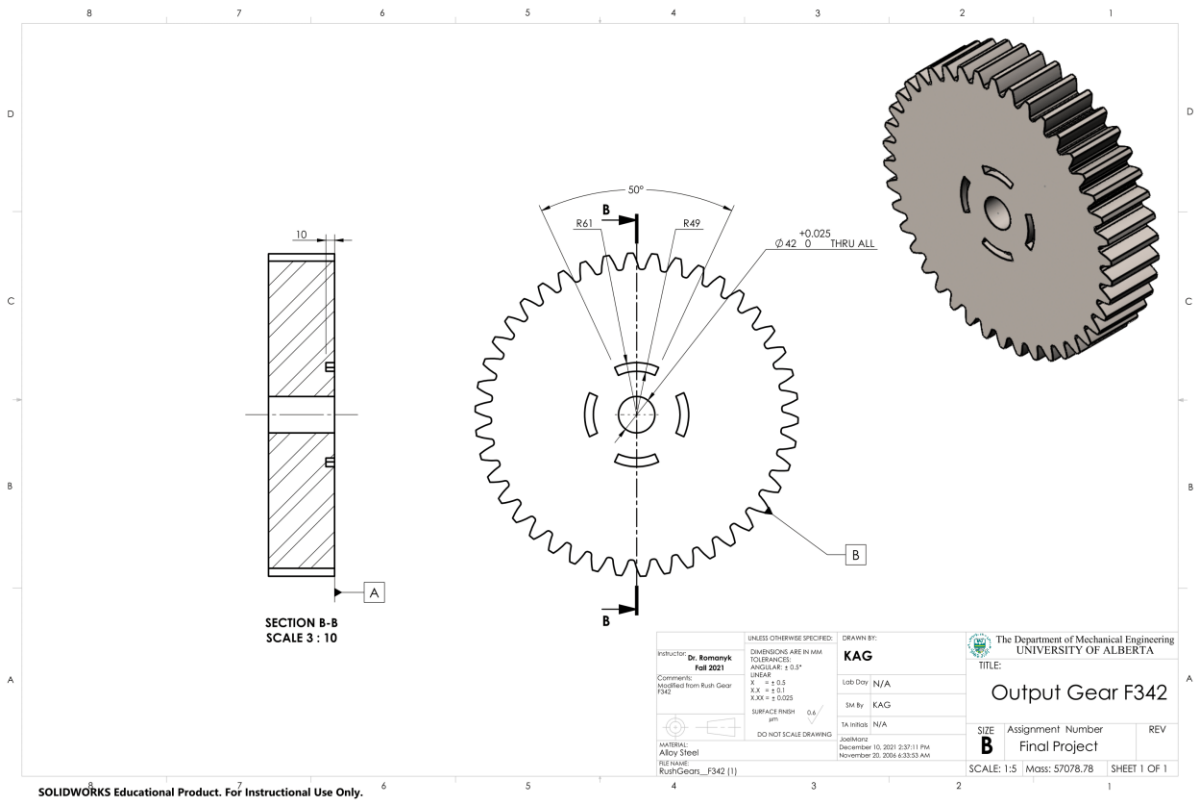
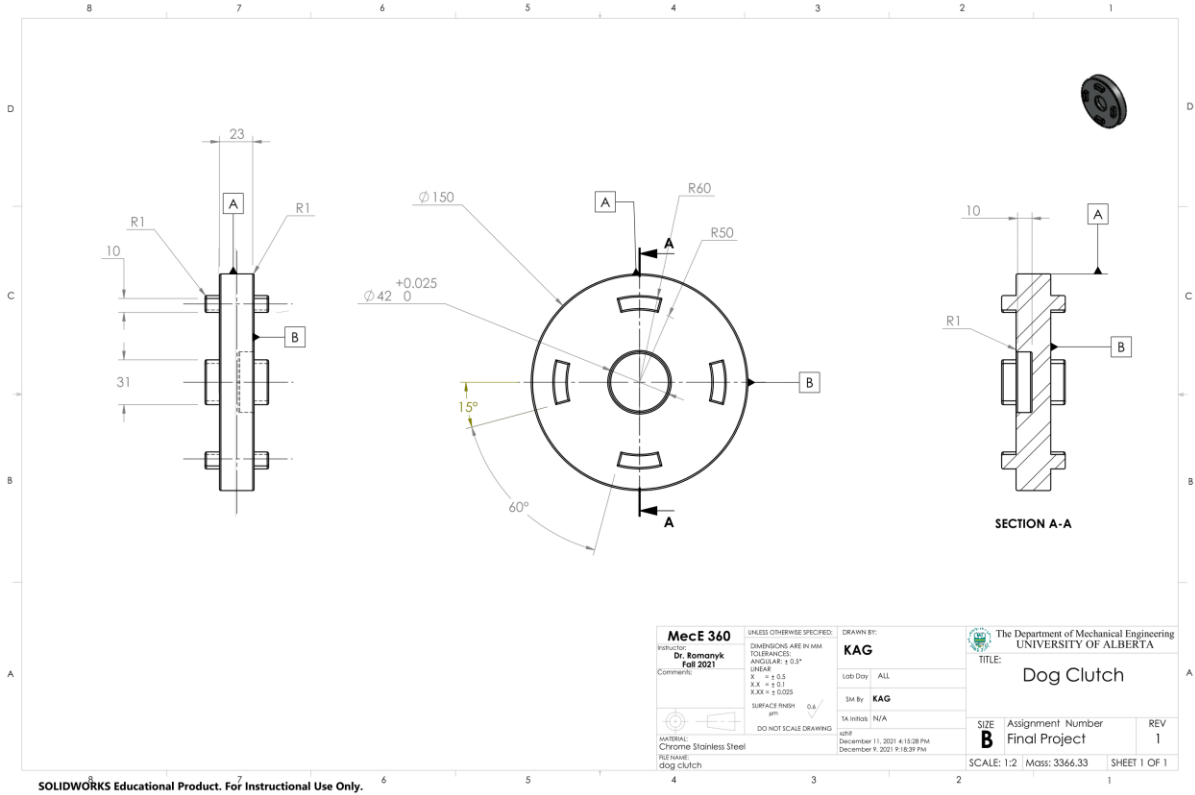
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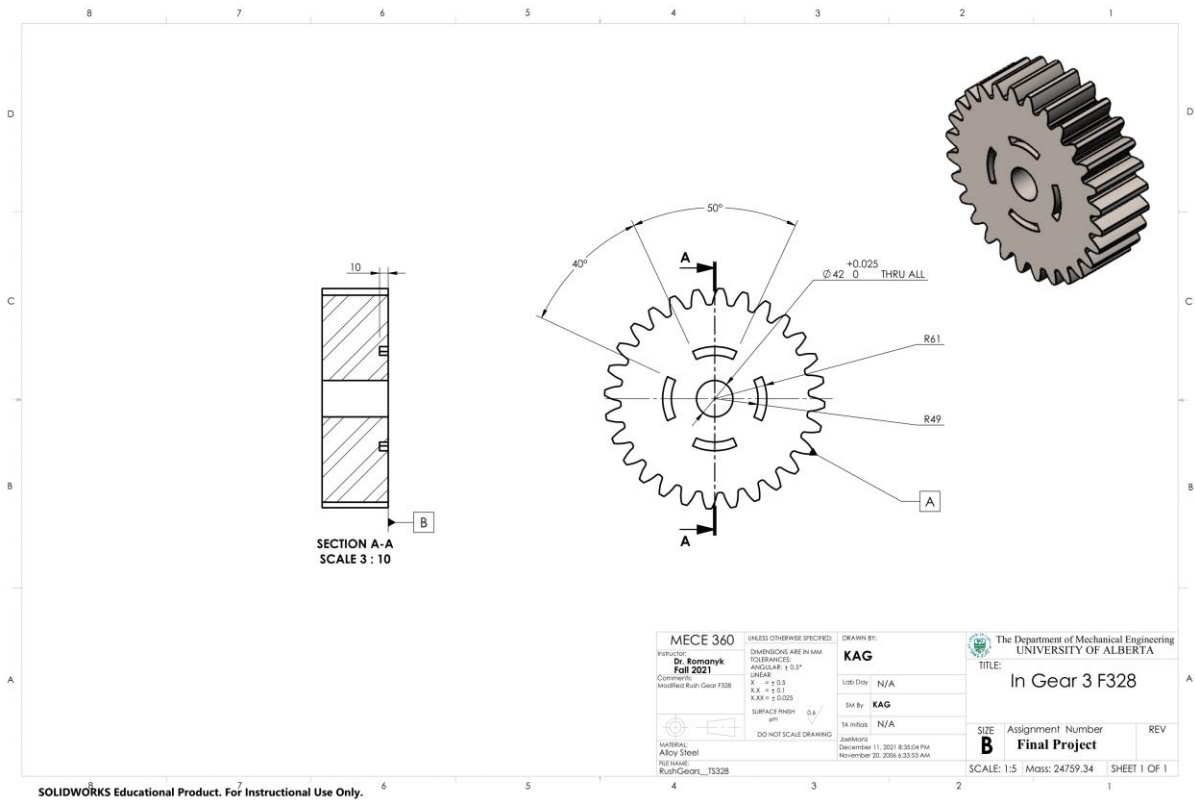
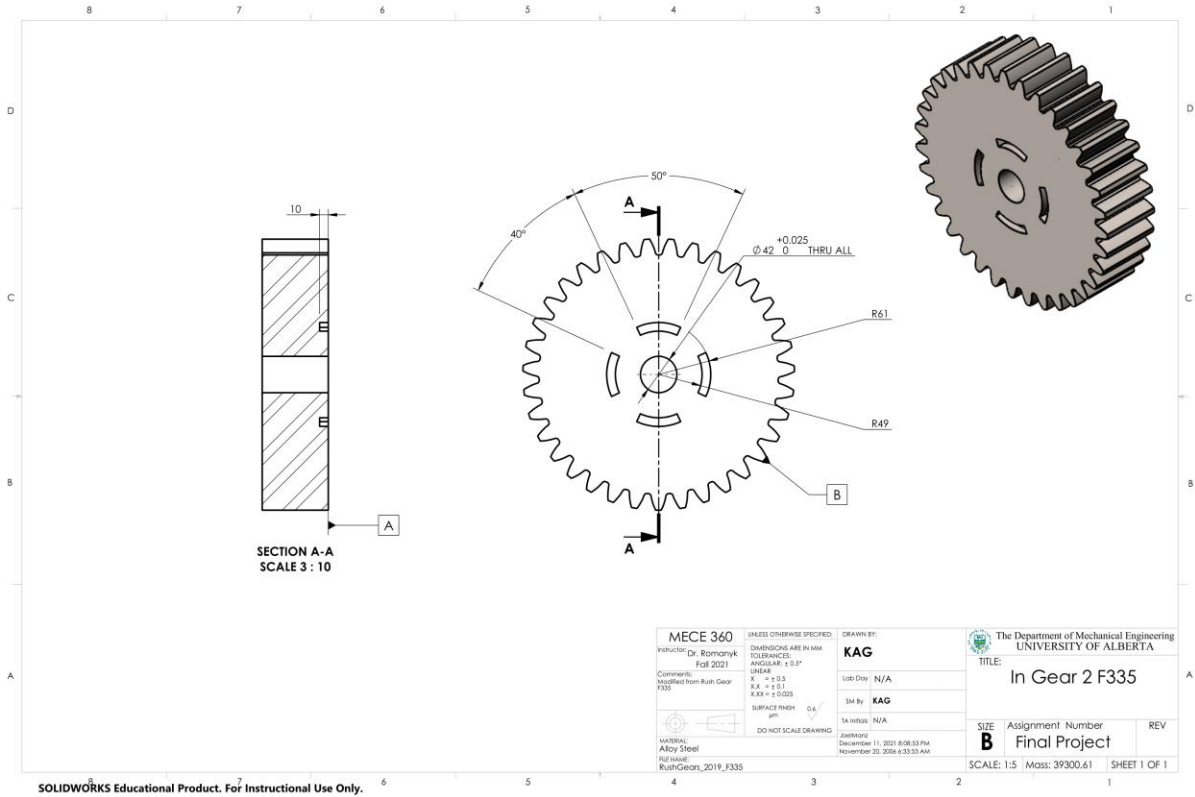
ITEM NO.	PART NUMBER	SW-Title	Mass	Material	Sw-Author	QTY.
1	Shaft	Input Shaft	4450.26	Alloy Steel	KAG	1
2	RushGears_2019_F321	Gear F321	13563.07	SCM415 Steel	RushGears	1
3	RushGears_TS328	Gear TS328	24434	SCM415 Steel	RushGears	1
4	RushGears_2019_F335	Gear F335	39409.27	SCM415 Steel	RushGears	1
5	RushGears_F342	Gear F342	57187.44	SCM415 Steel	RushGears	1
6	7907ASTYN	40mm bearing	0.074	Chrome Stainless Steel	NSK	2
7	shifter shaft	Shifter Shaft	25260.32	AISI 1020	KAG	1
8	dog clutch	Dog Clutch	3225.41	Chrome Stainless Steel	KAG	2
9	shifterlink	Shifter link	949.87	Alloy Steel	KAG	2

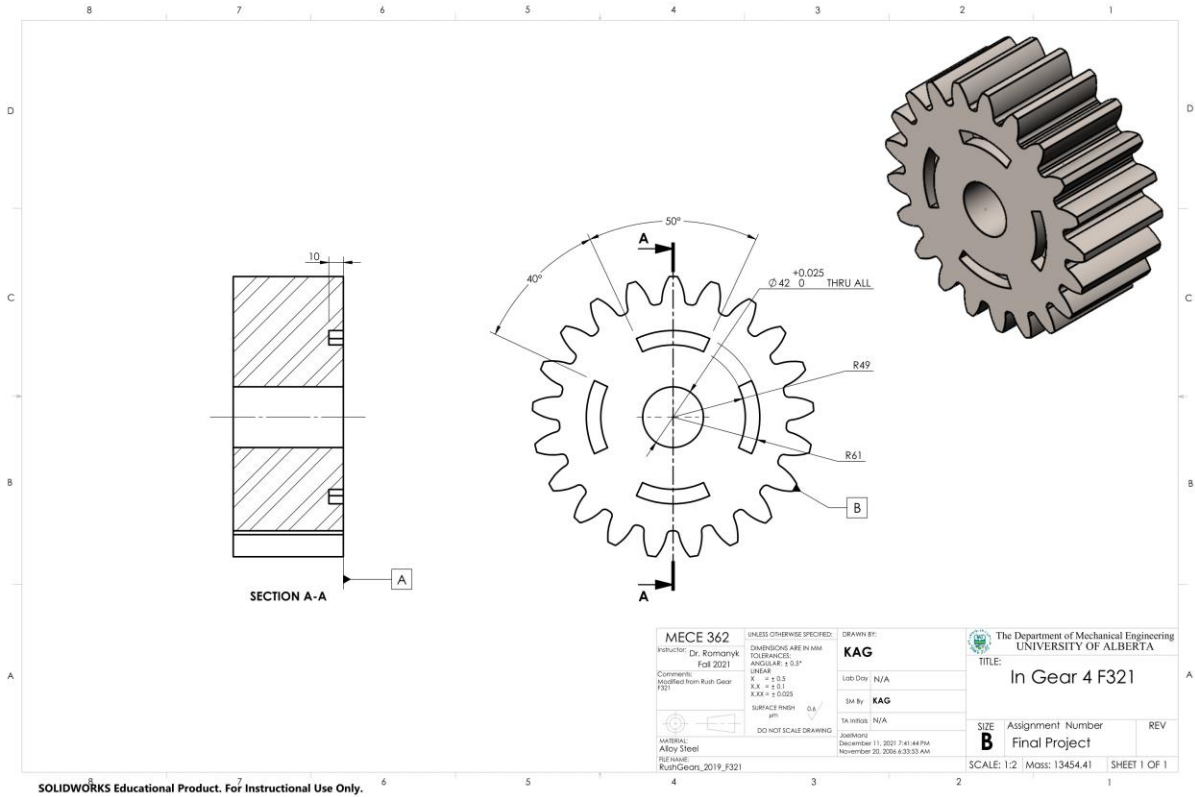


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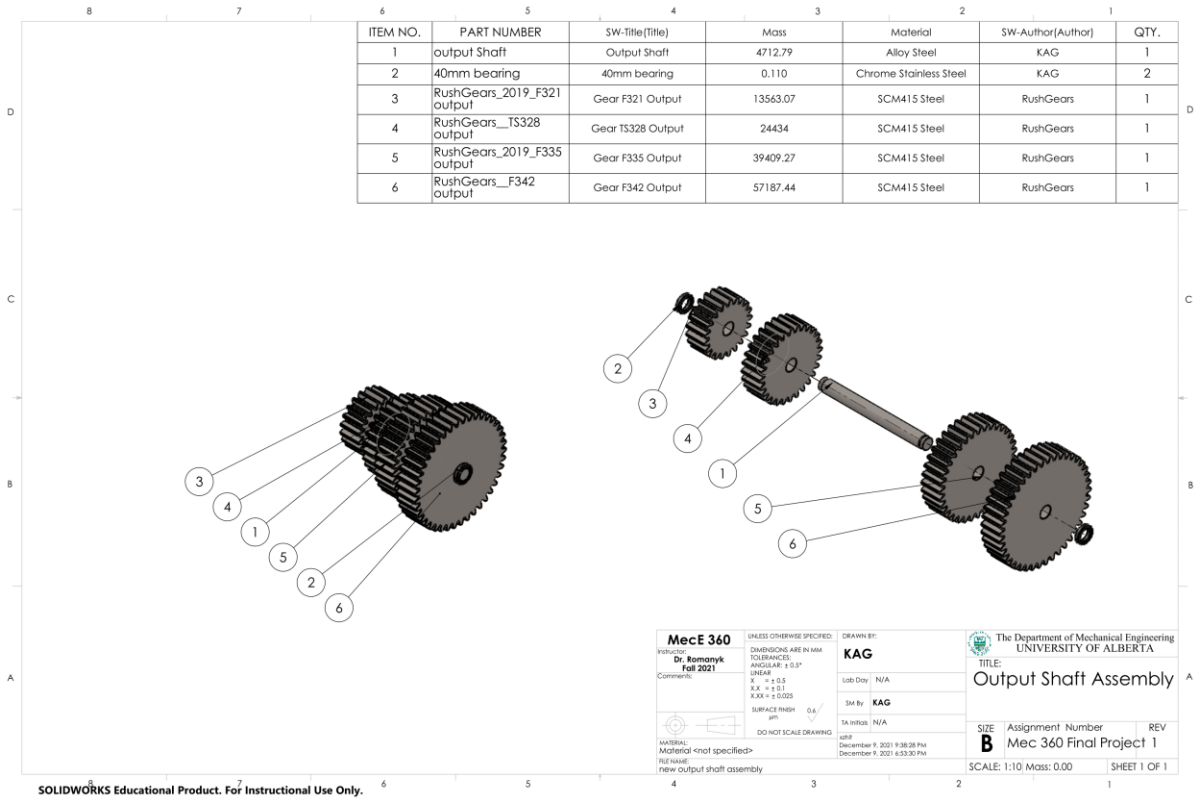




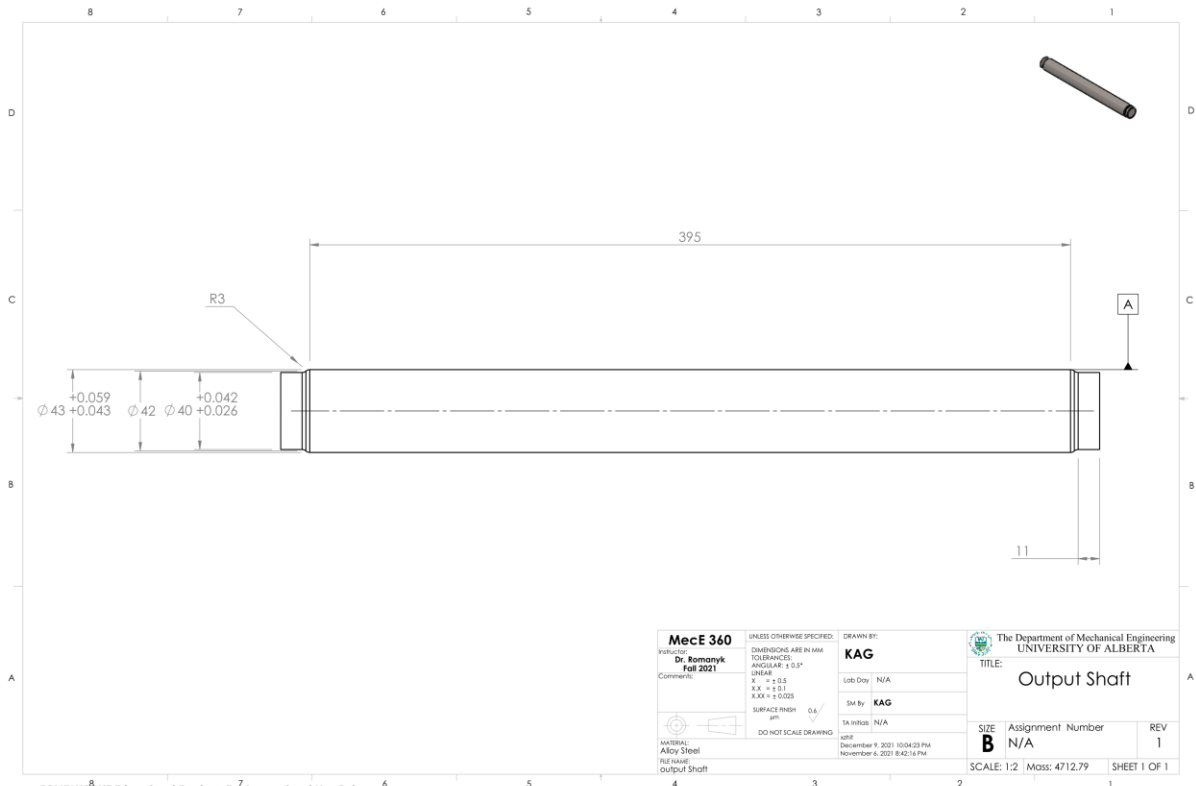




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